Collisions for RC4-Hash

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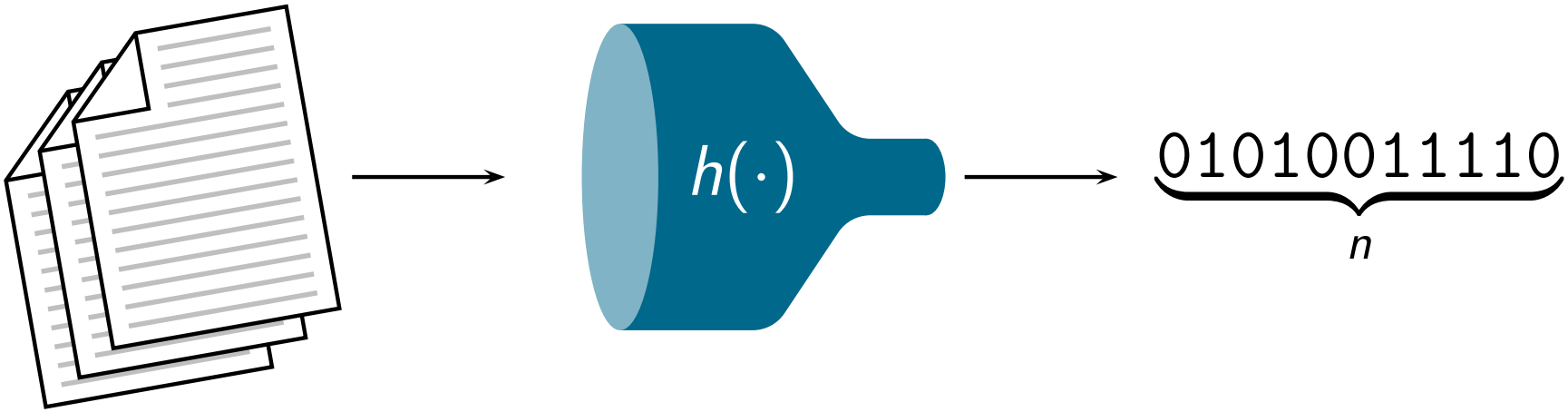
Outline

1. **Introduction**
   - Cryptographic Hash Functions
   - RC4-Hash
2. **Fixed Points for RC4-Hash**
   - Type I
   - Type II
3. **Collisions for RC4-Hash**
4. **Conclusion**

Cryptographic Hash Functions



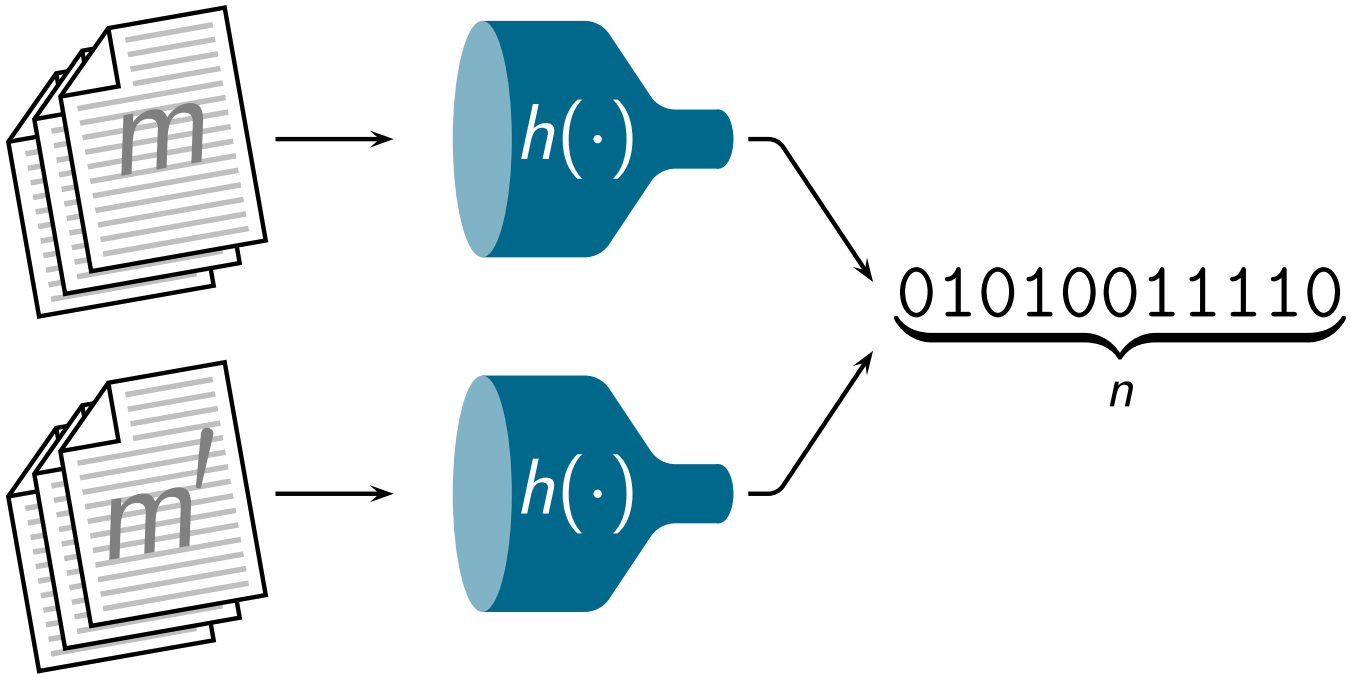
$$h : \{0,1\}^* \mapsto \{0,1\}^n$$

Desired properties

- **Collision resistance**
- (Second) preimage resistance
- ...

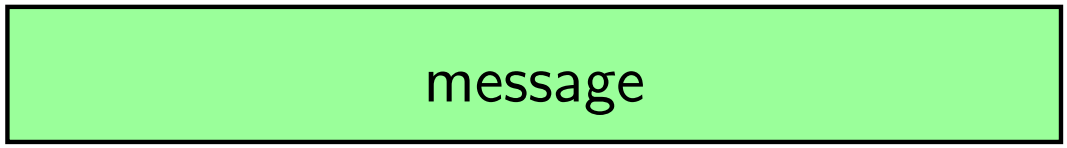
Cryptographic Hash Functions

Collision Resistance



- "Hard" to find $m \neq m'$ s.t. $h(m) = h(m')$.
- Birthday paradox $\mathcal{O}(2^{n/2})$
- This talk: **RC4-Hash is not collision resistant**

RC4-Hash



RC4-Hash

- D. Chang, K. C. Gupta, and M. Nandi (2006)

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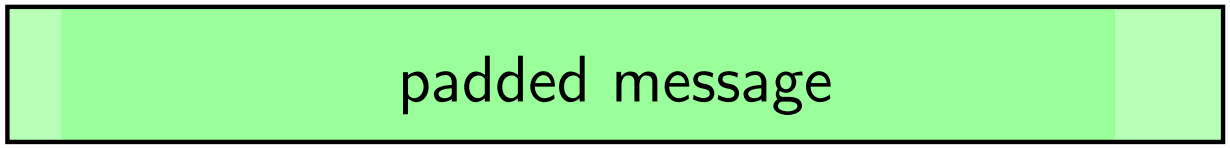

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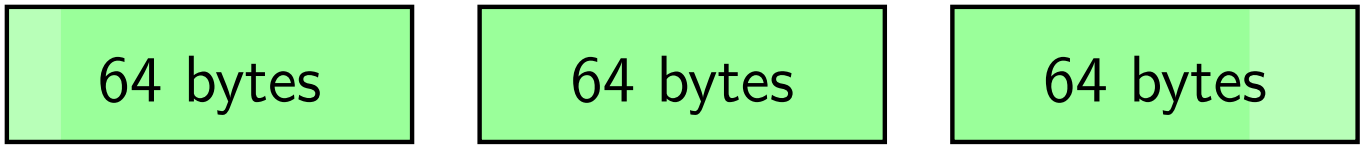

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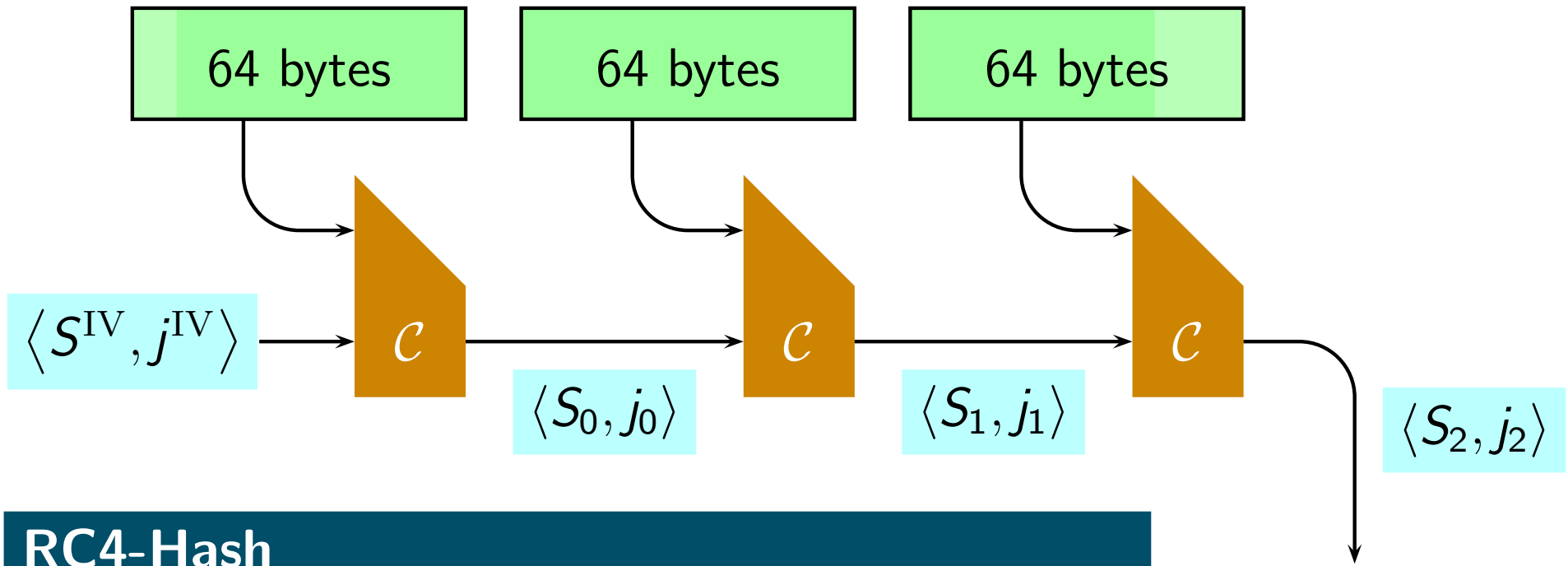

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- Chaining value $\langle S, j \rangle$ (cf. RC4 state)

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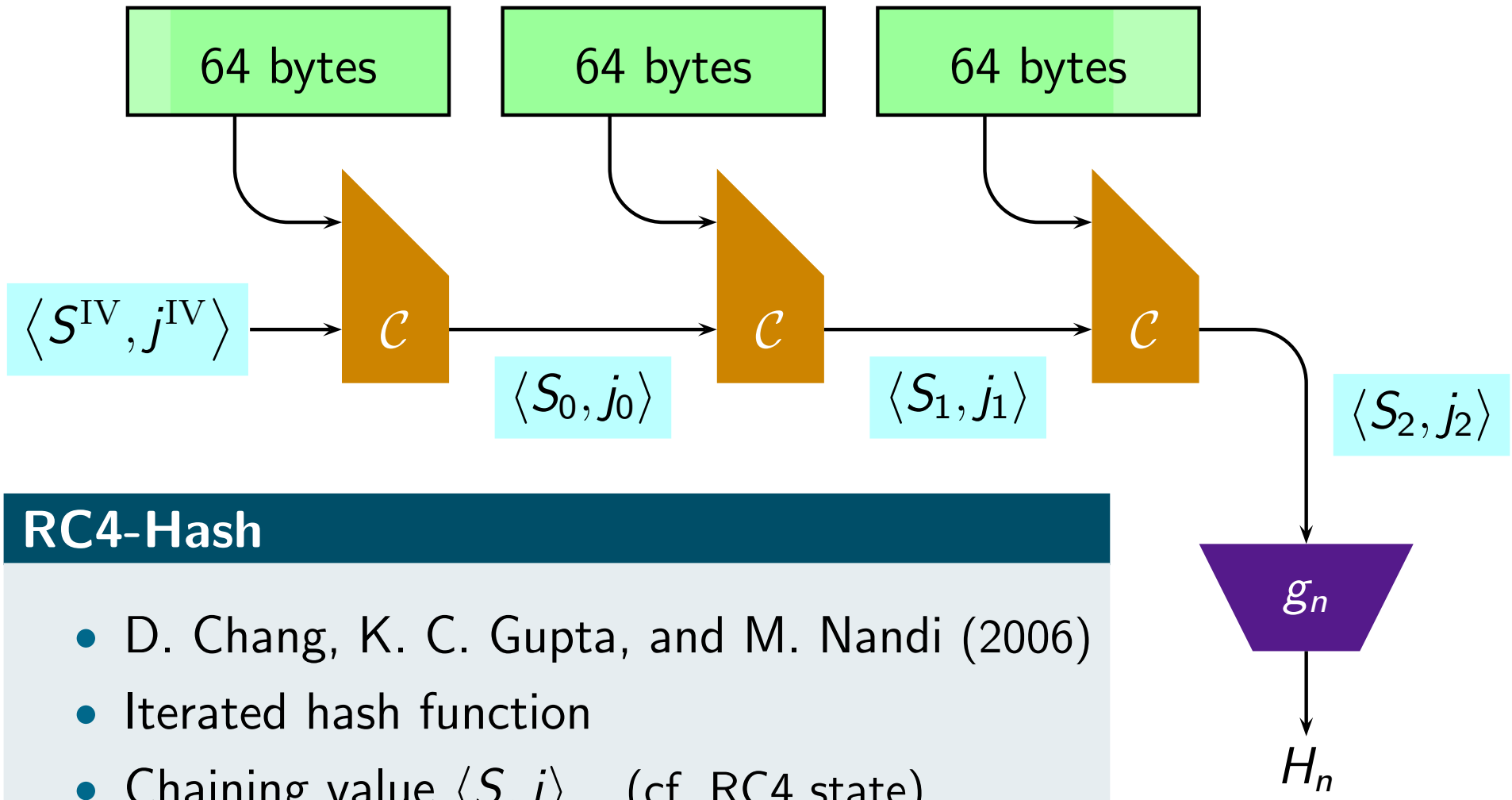

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- Iterated hash function
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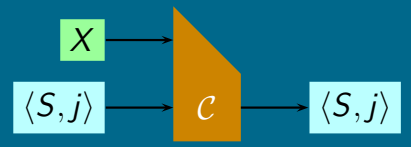
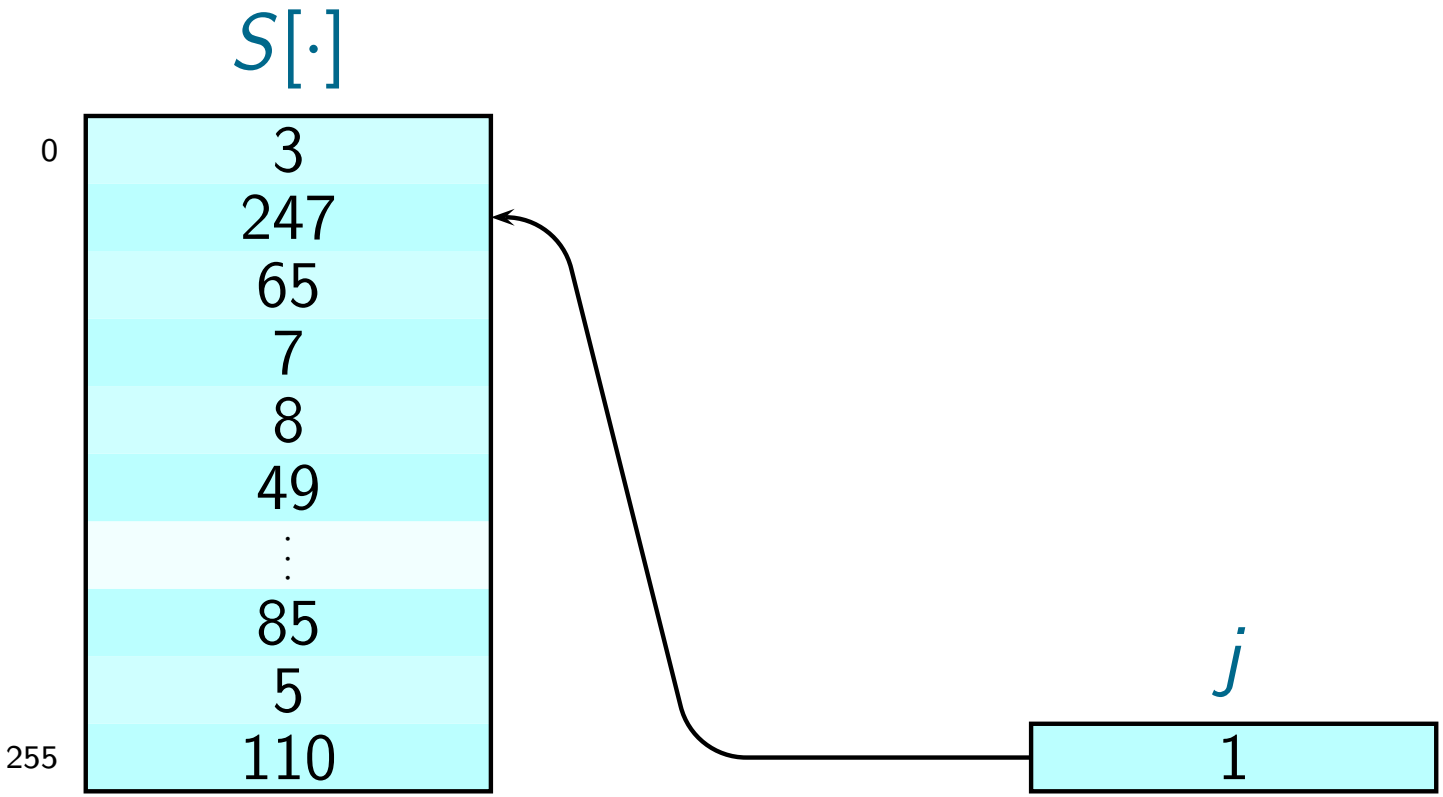
- Variable output length $n = 128 - 512$bits

RC4-Hash

Compression Function





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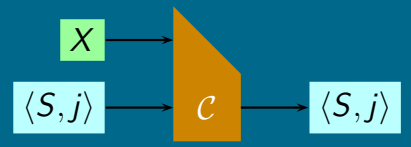

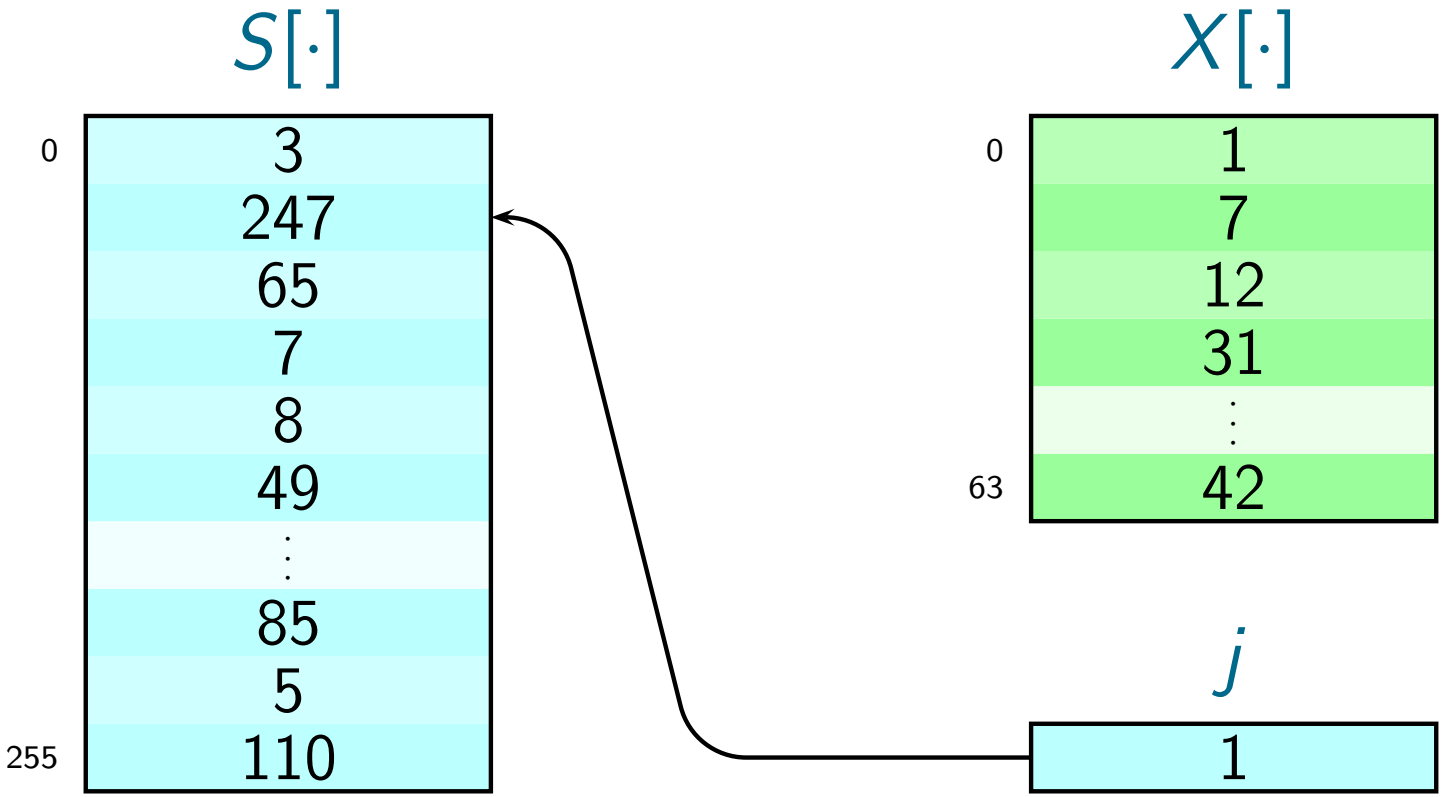

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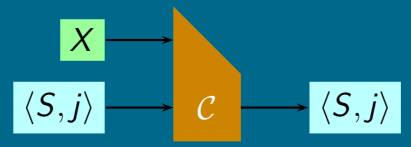

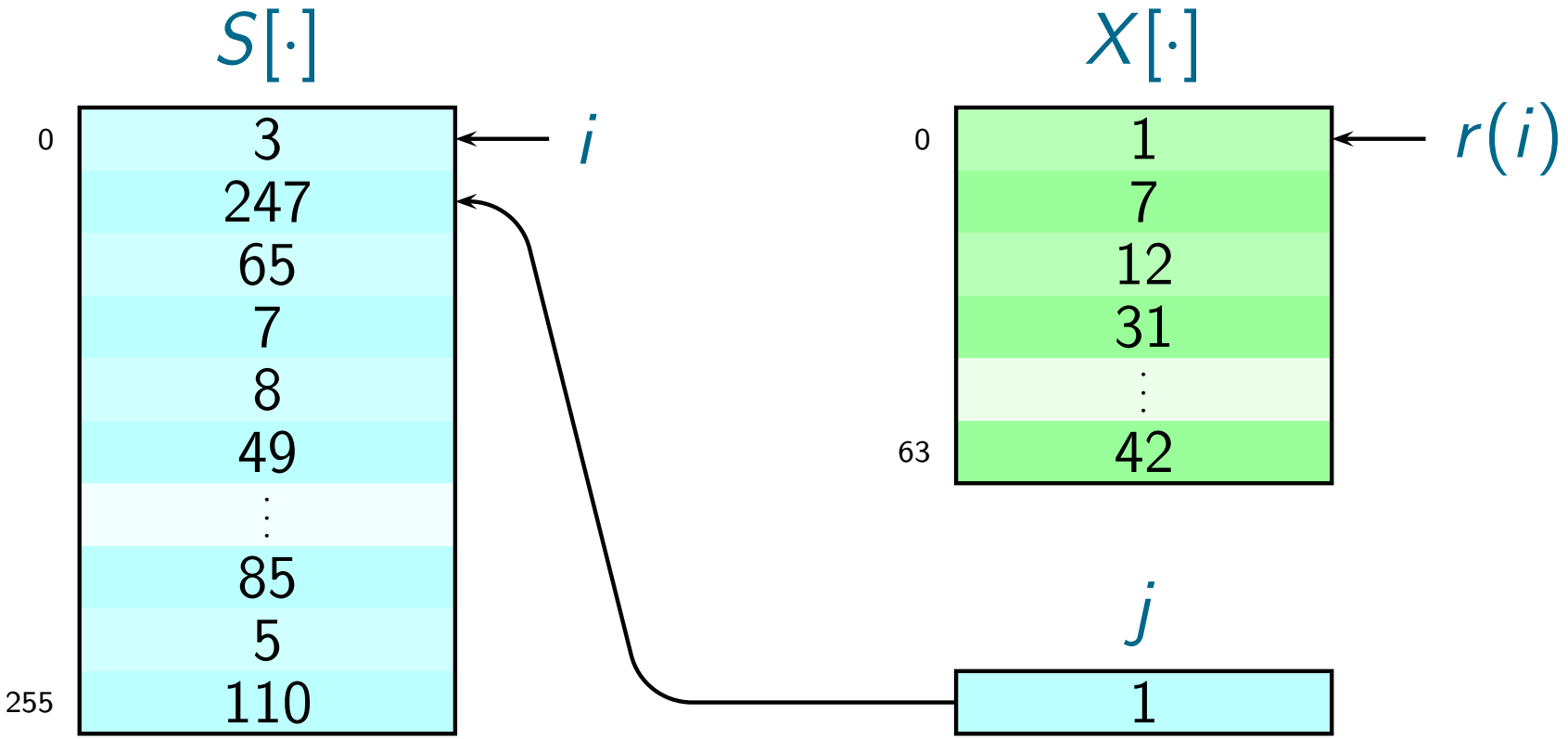

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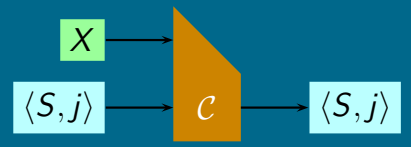

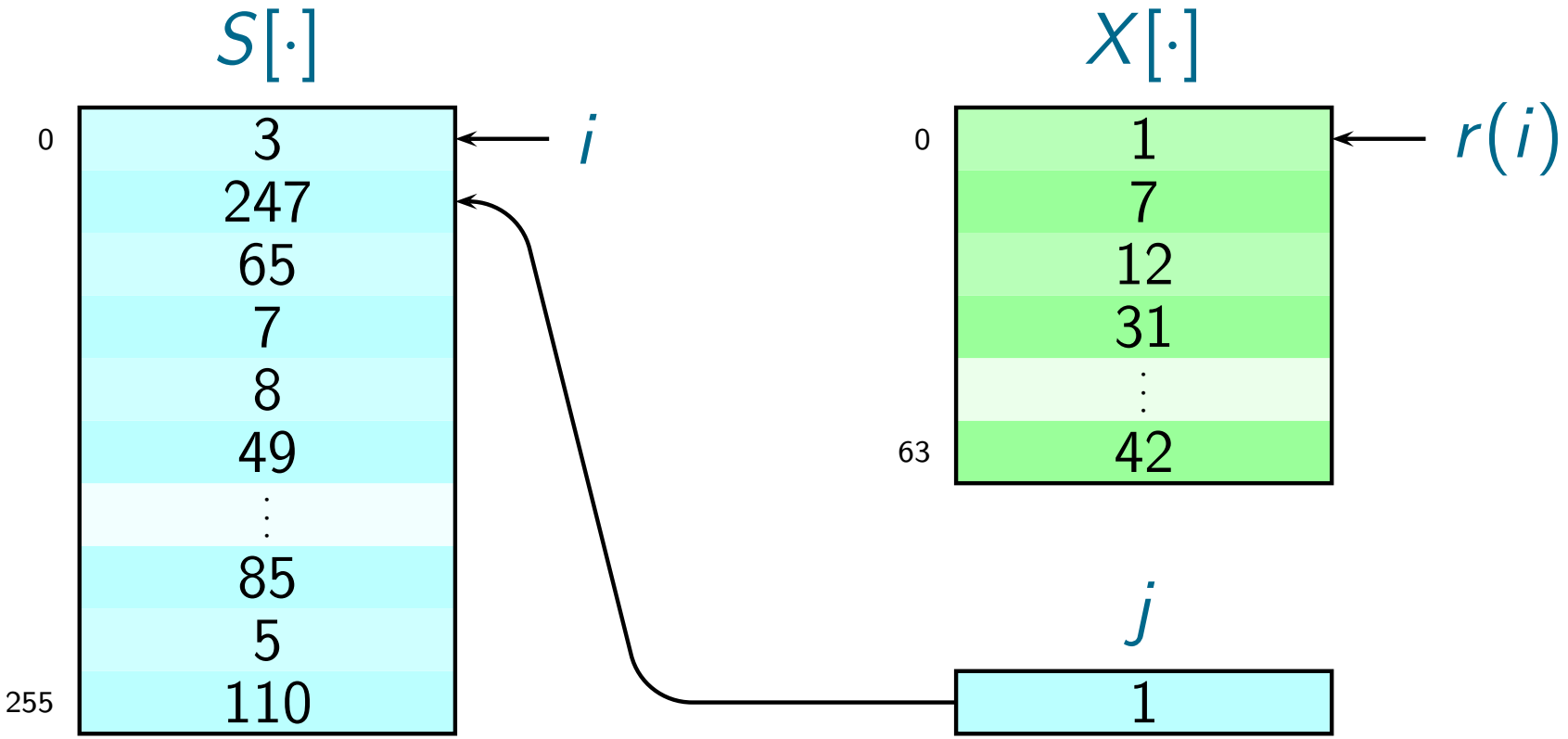

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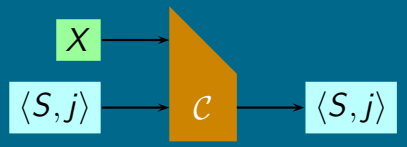

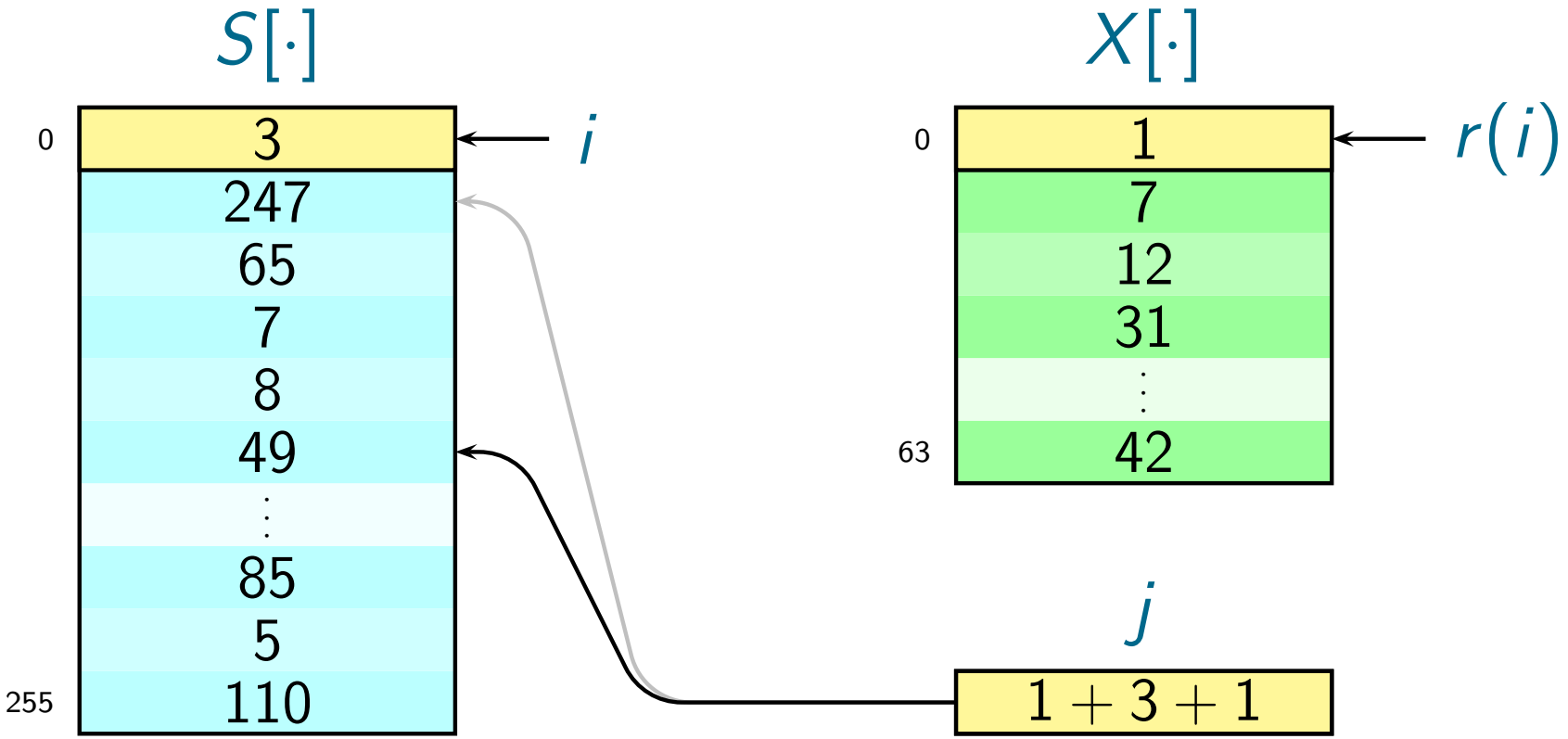

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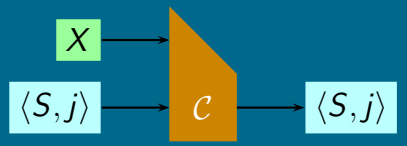

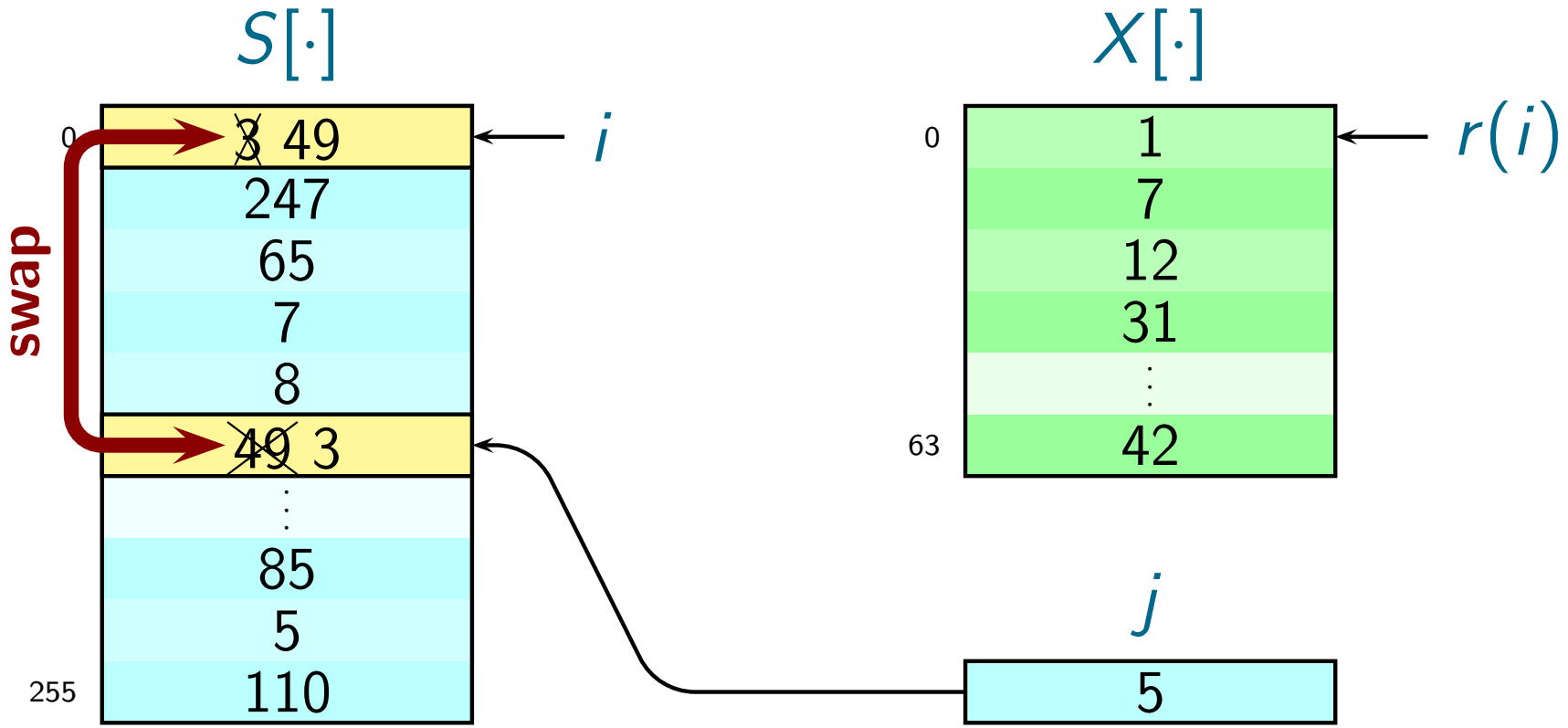

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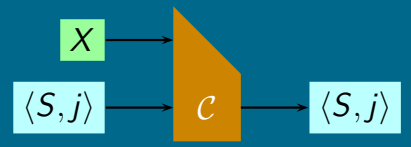

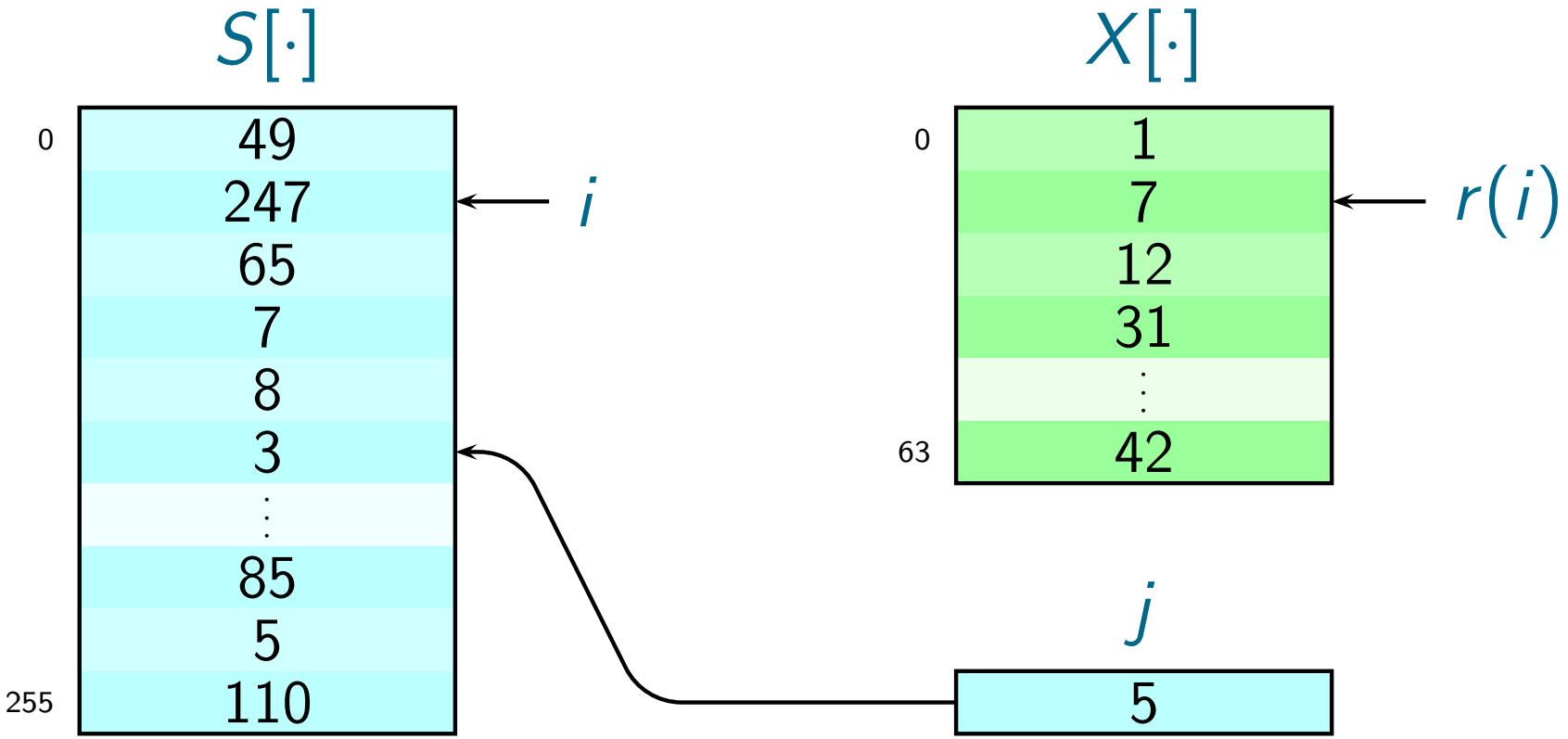

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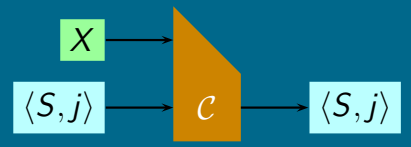

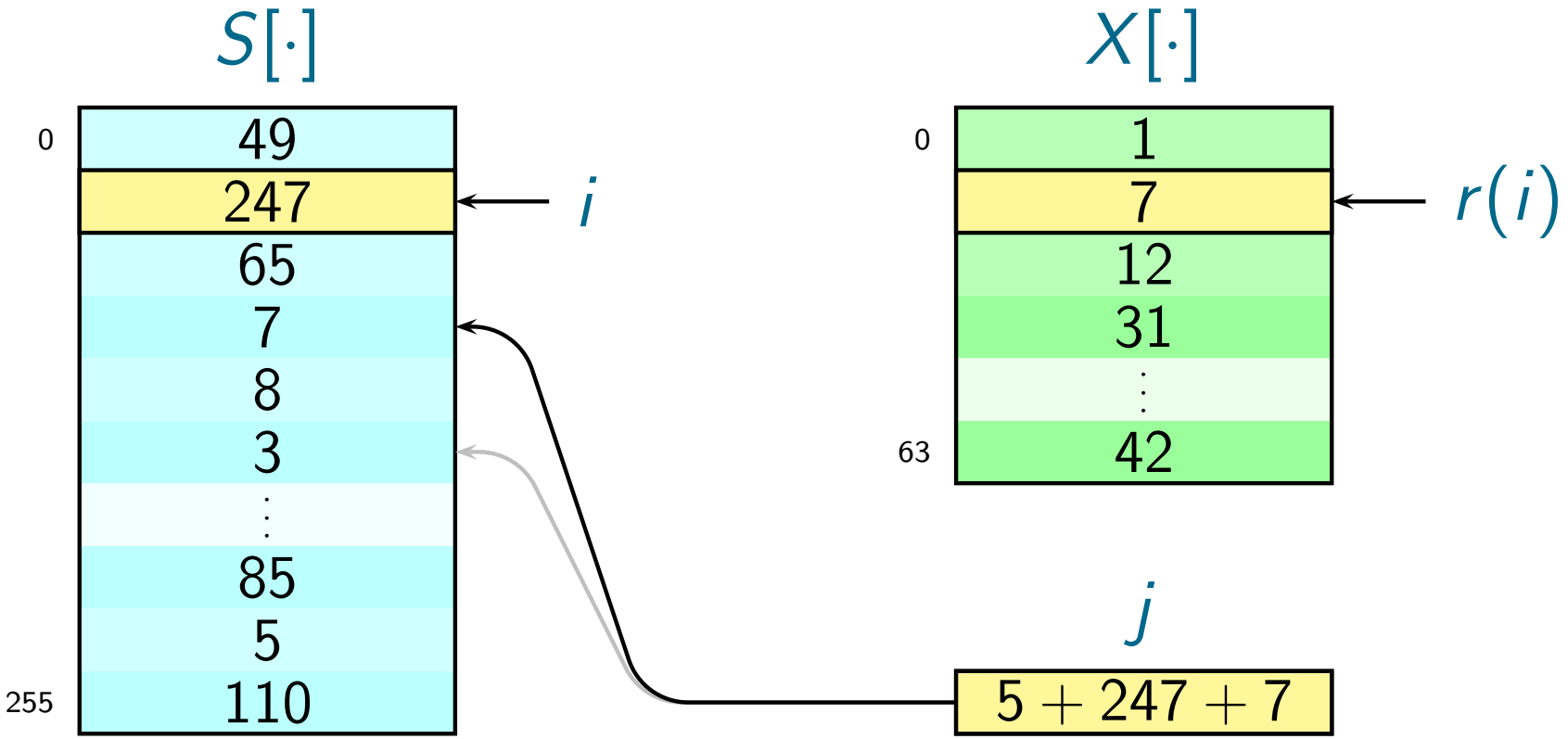

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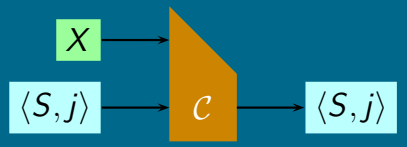

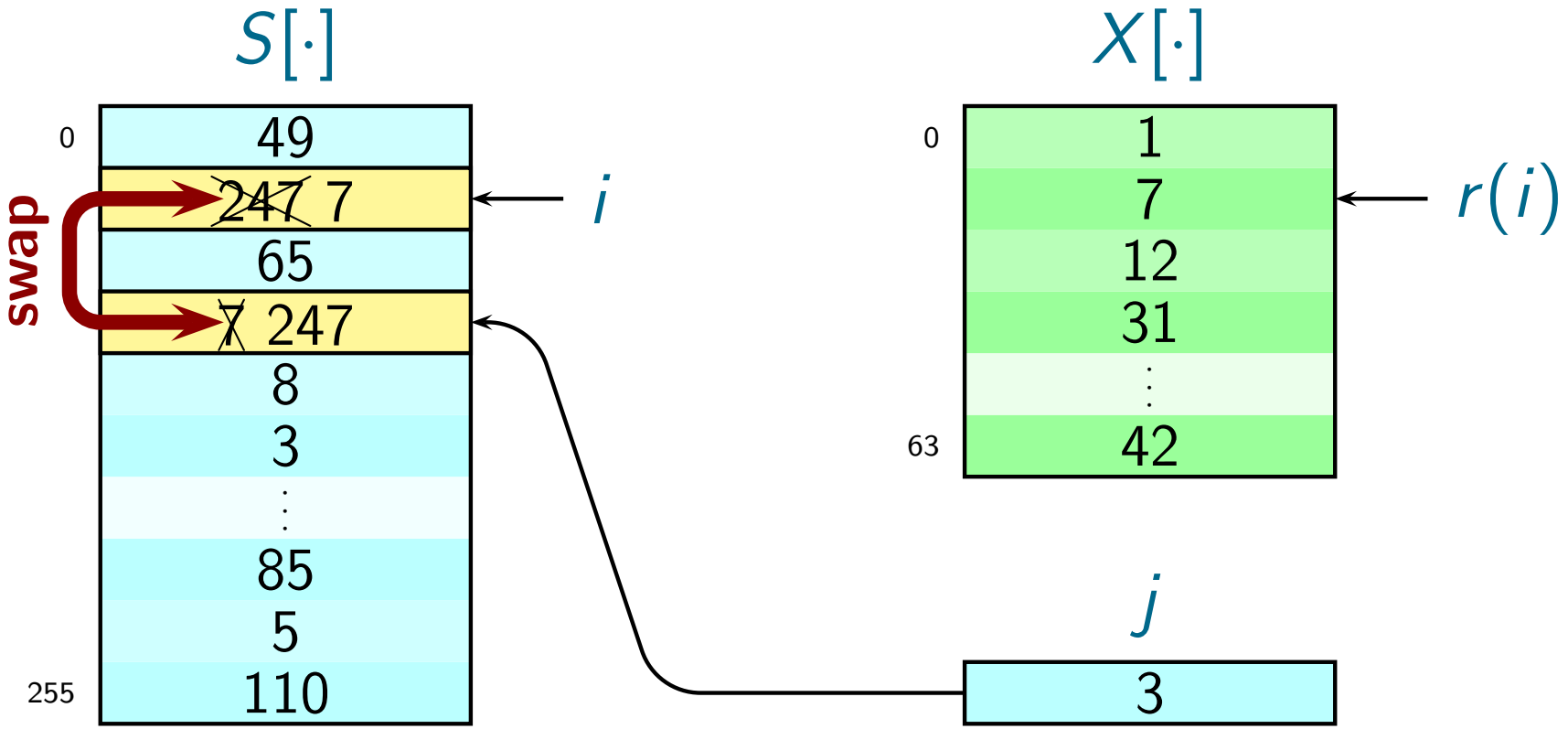

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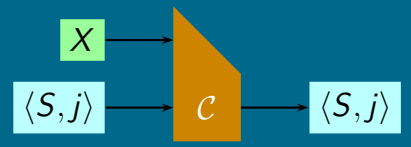

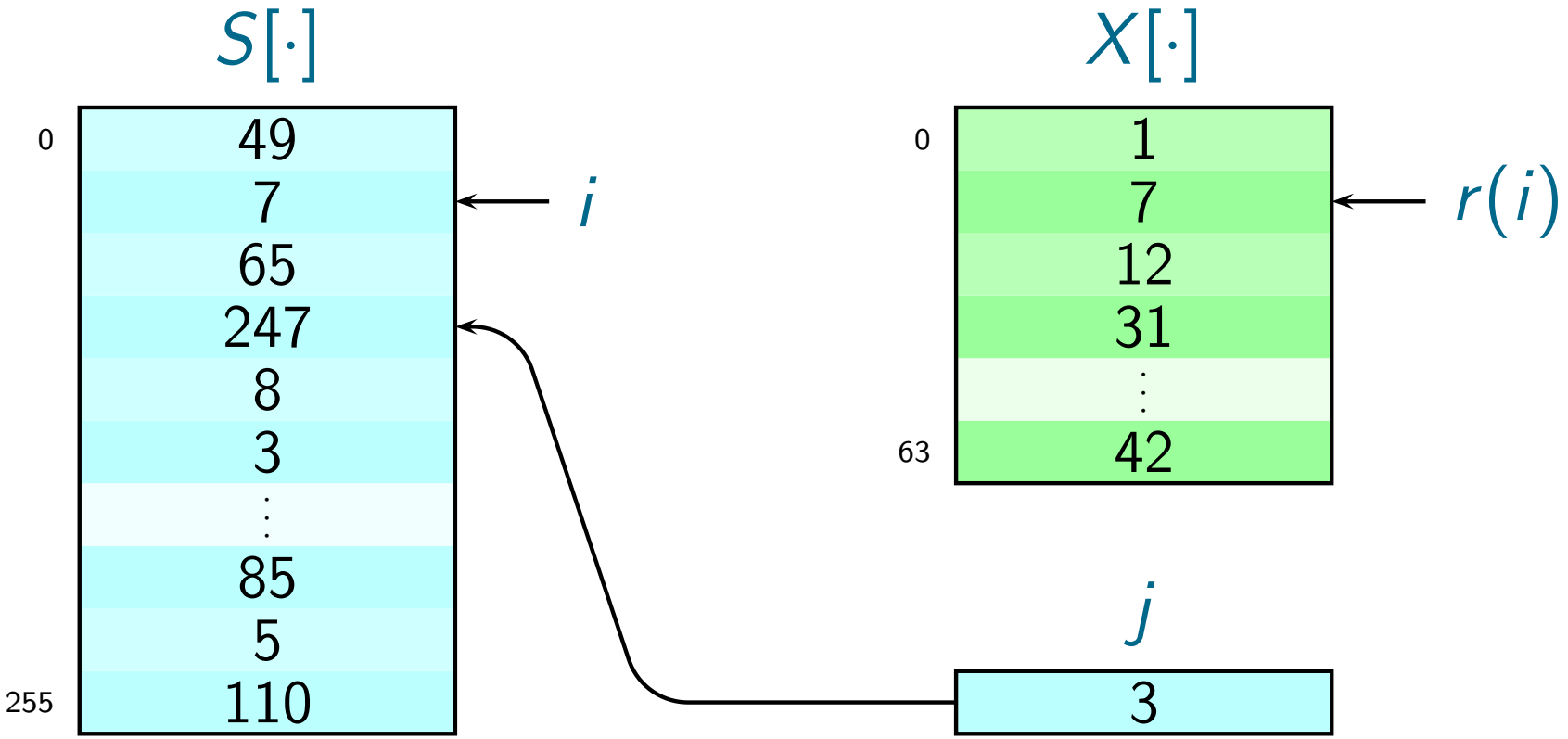

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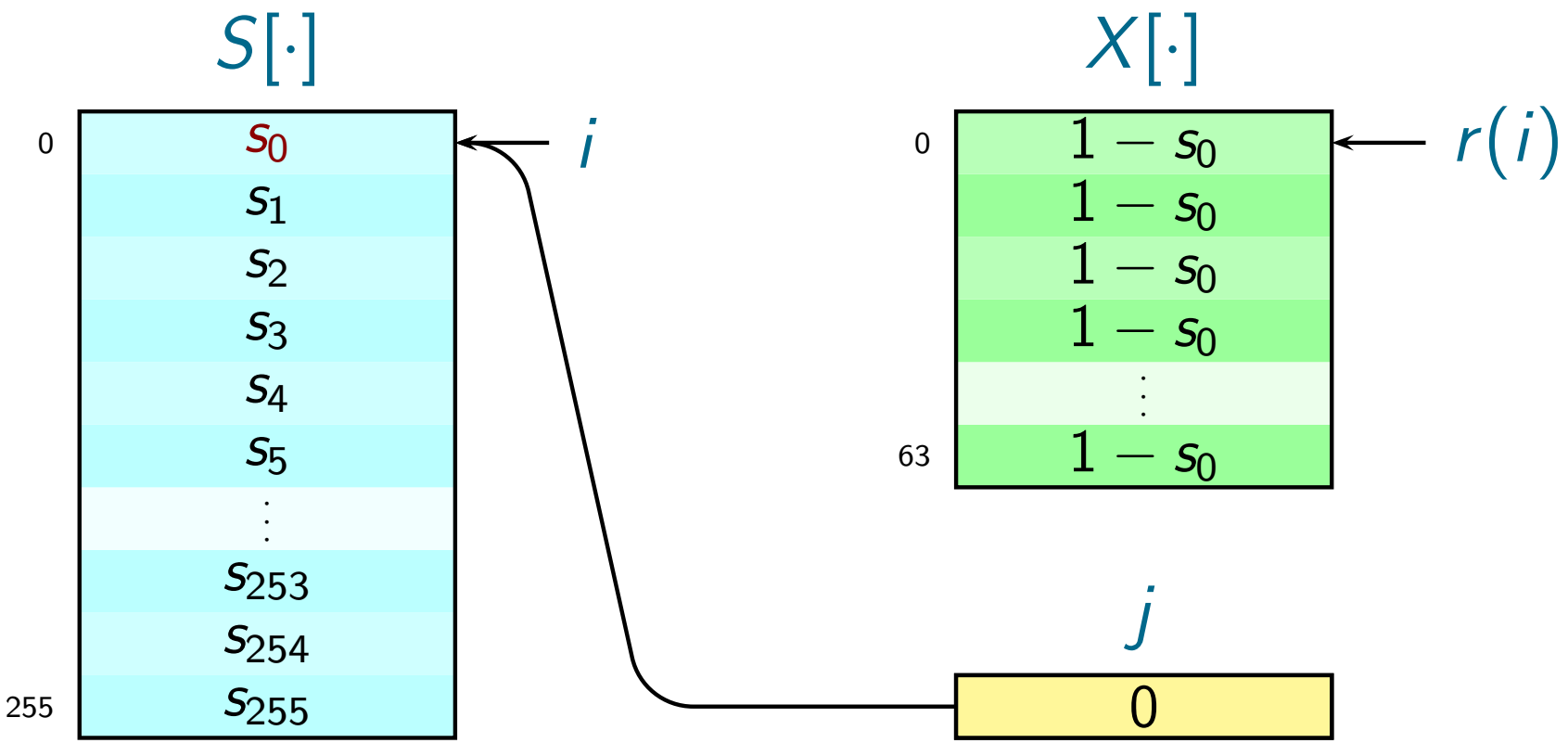

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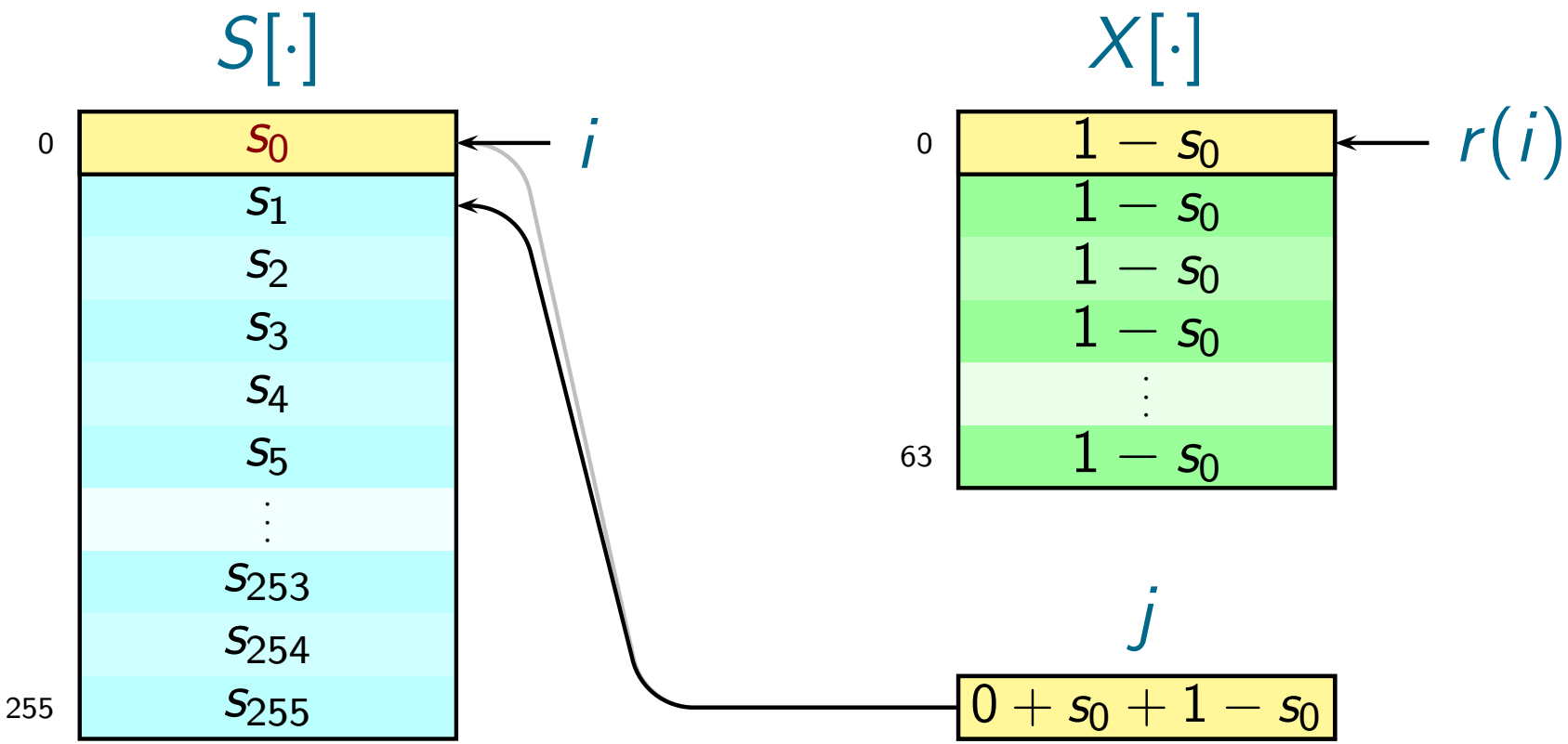

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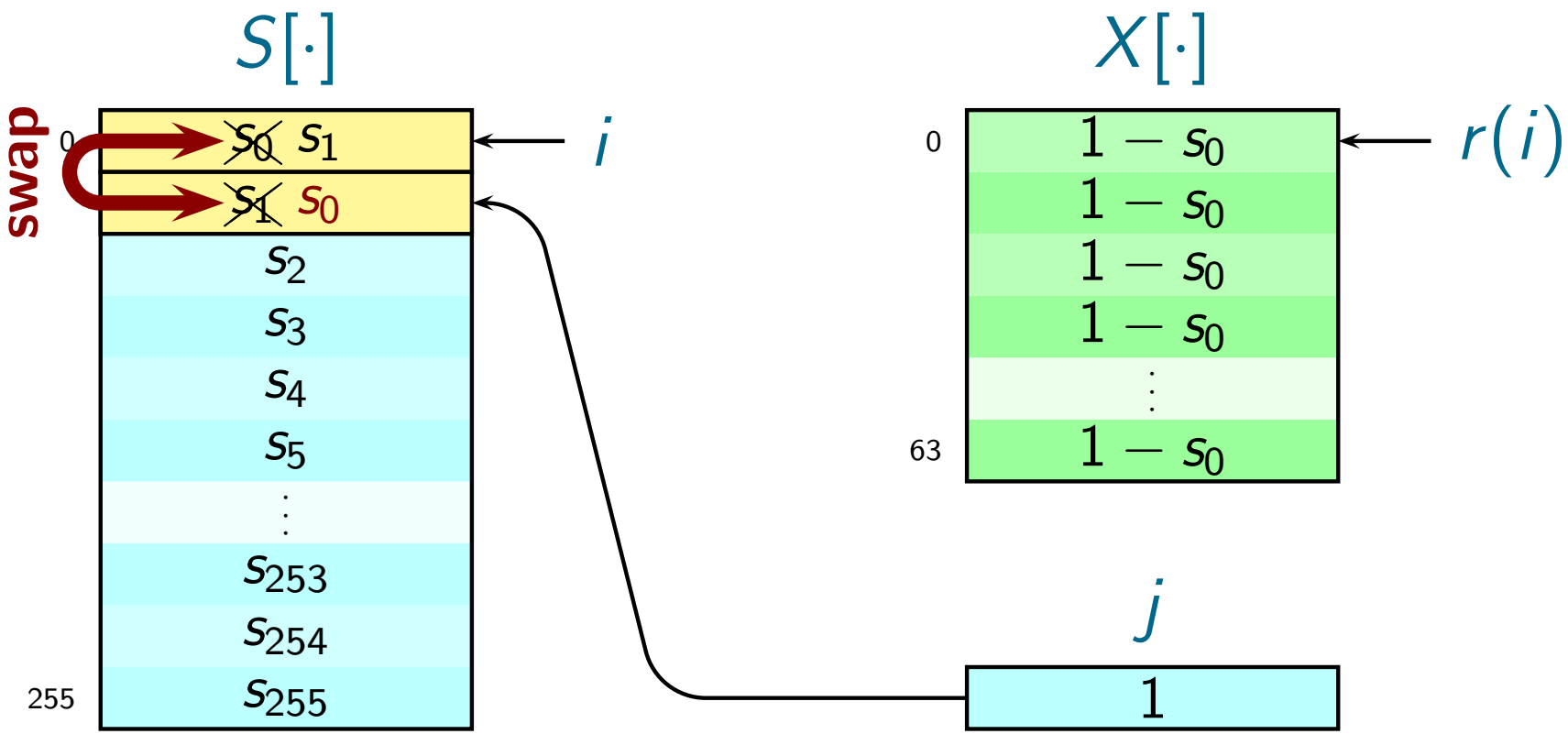

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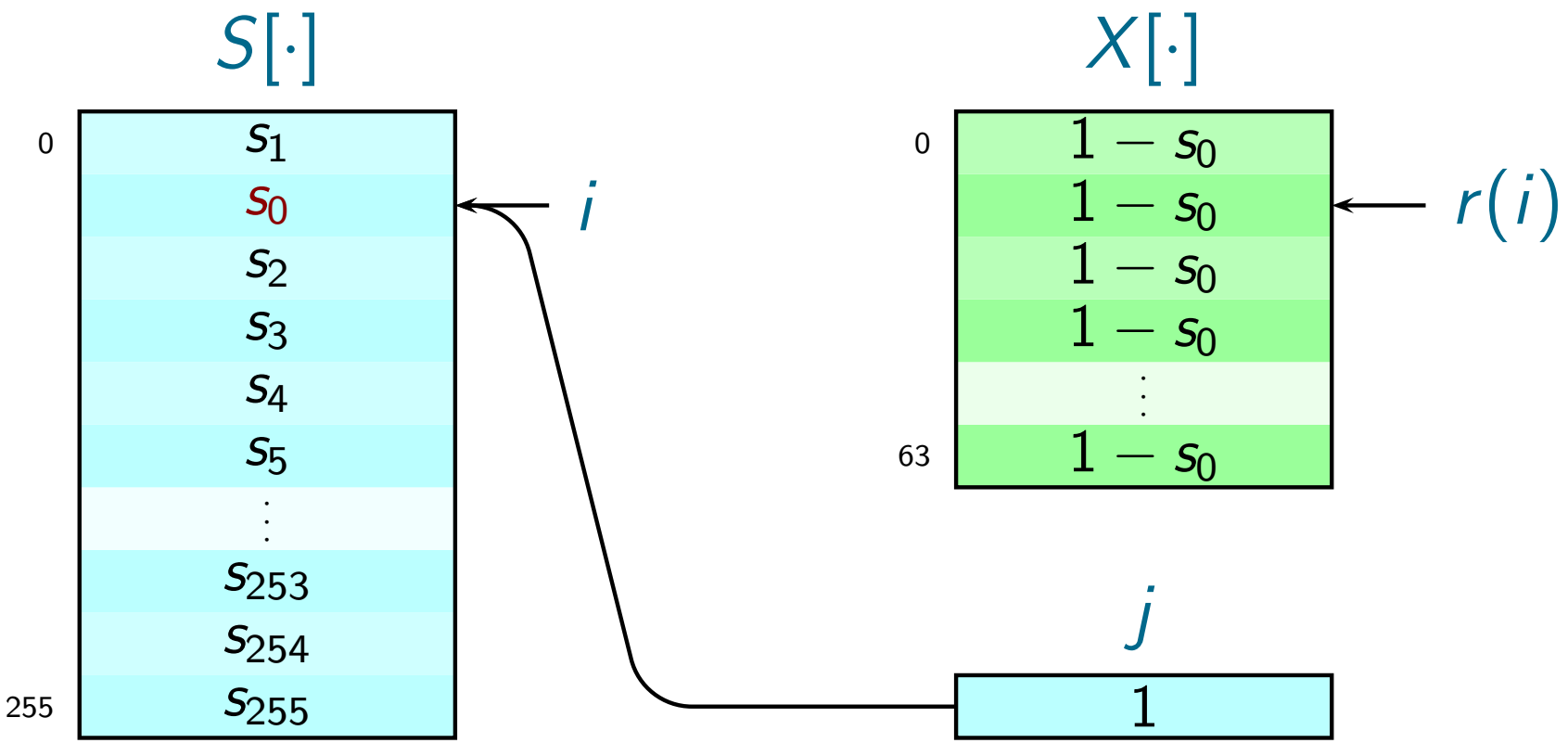

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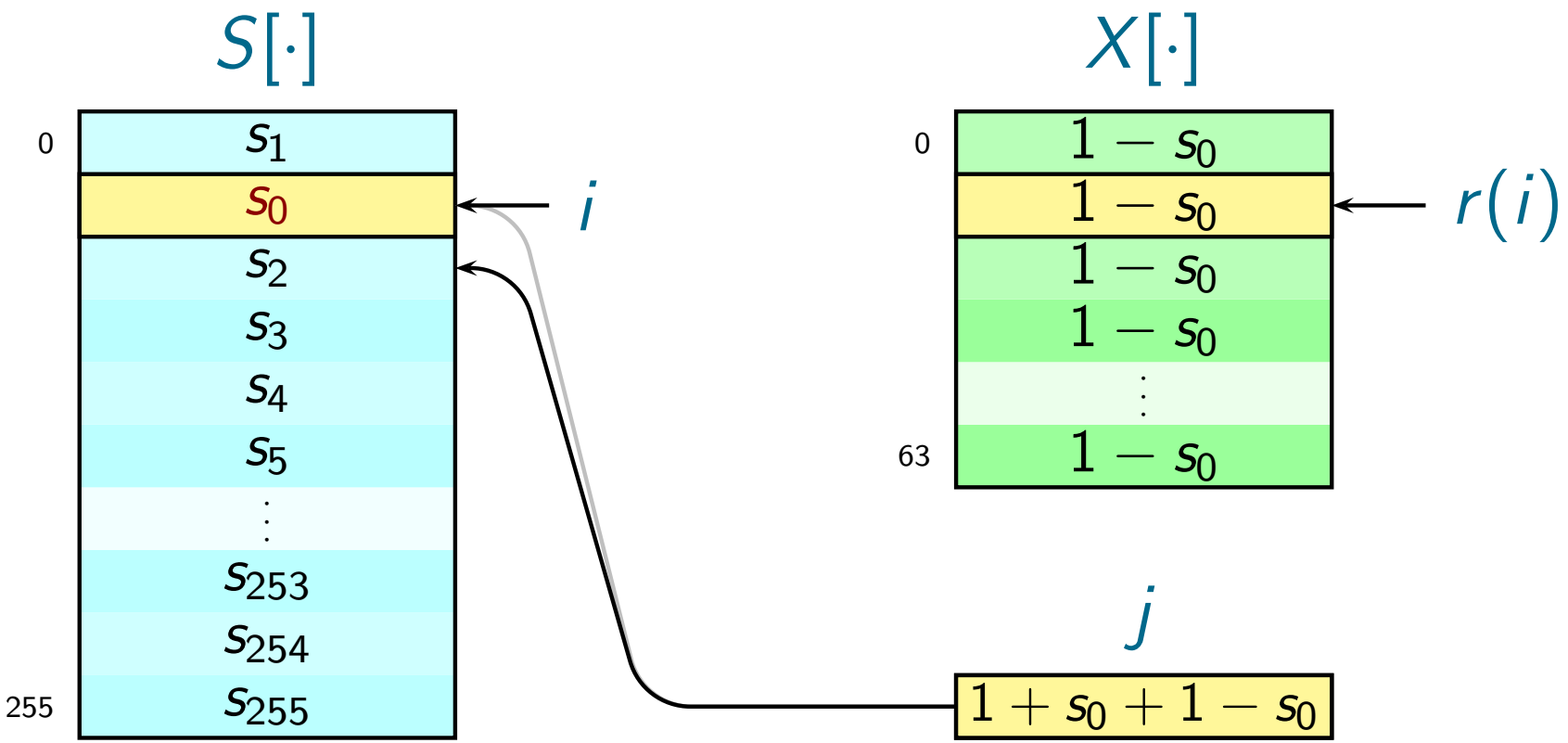

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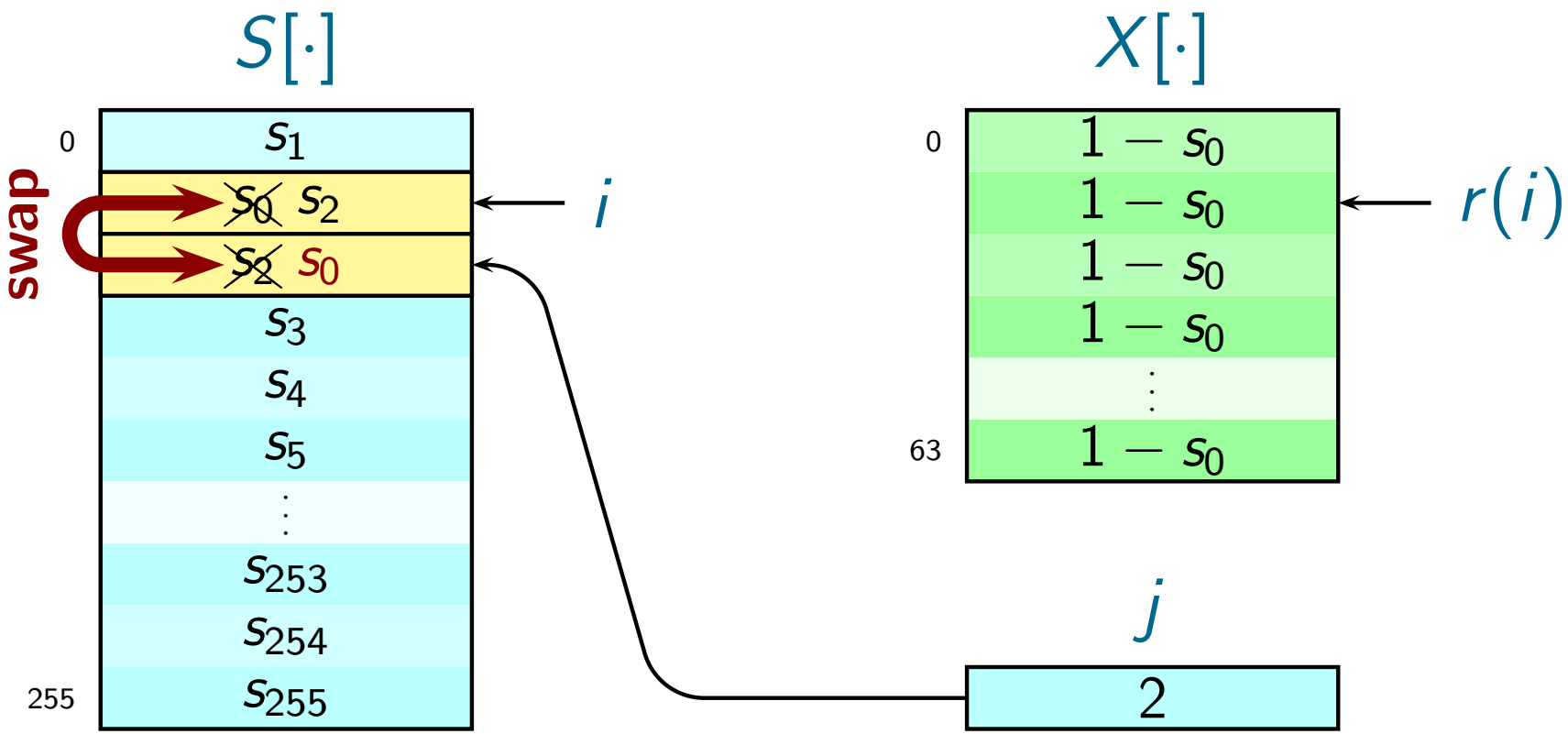

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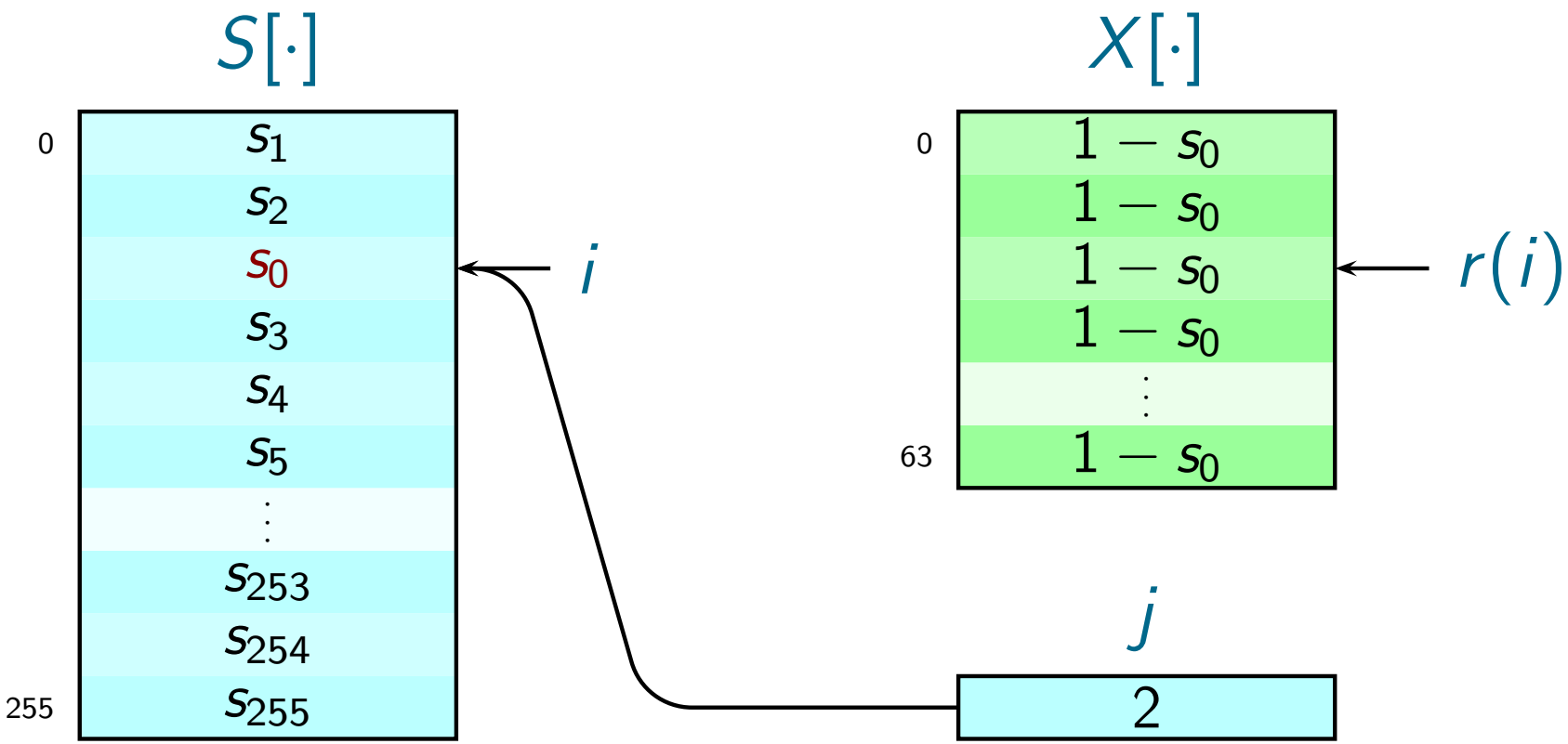

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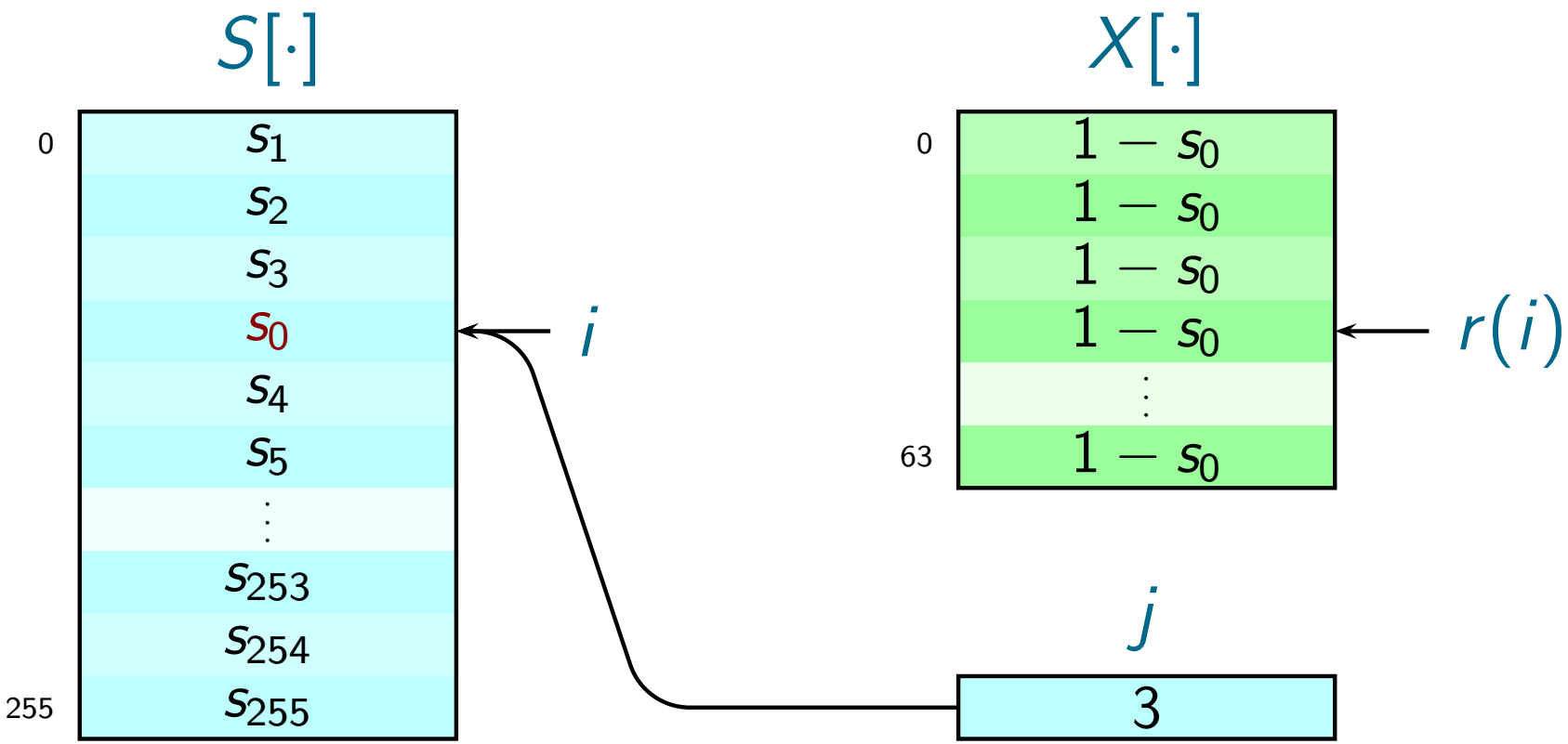

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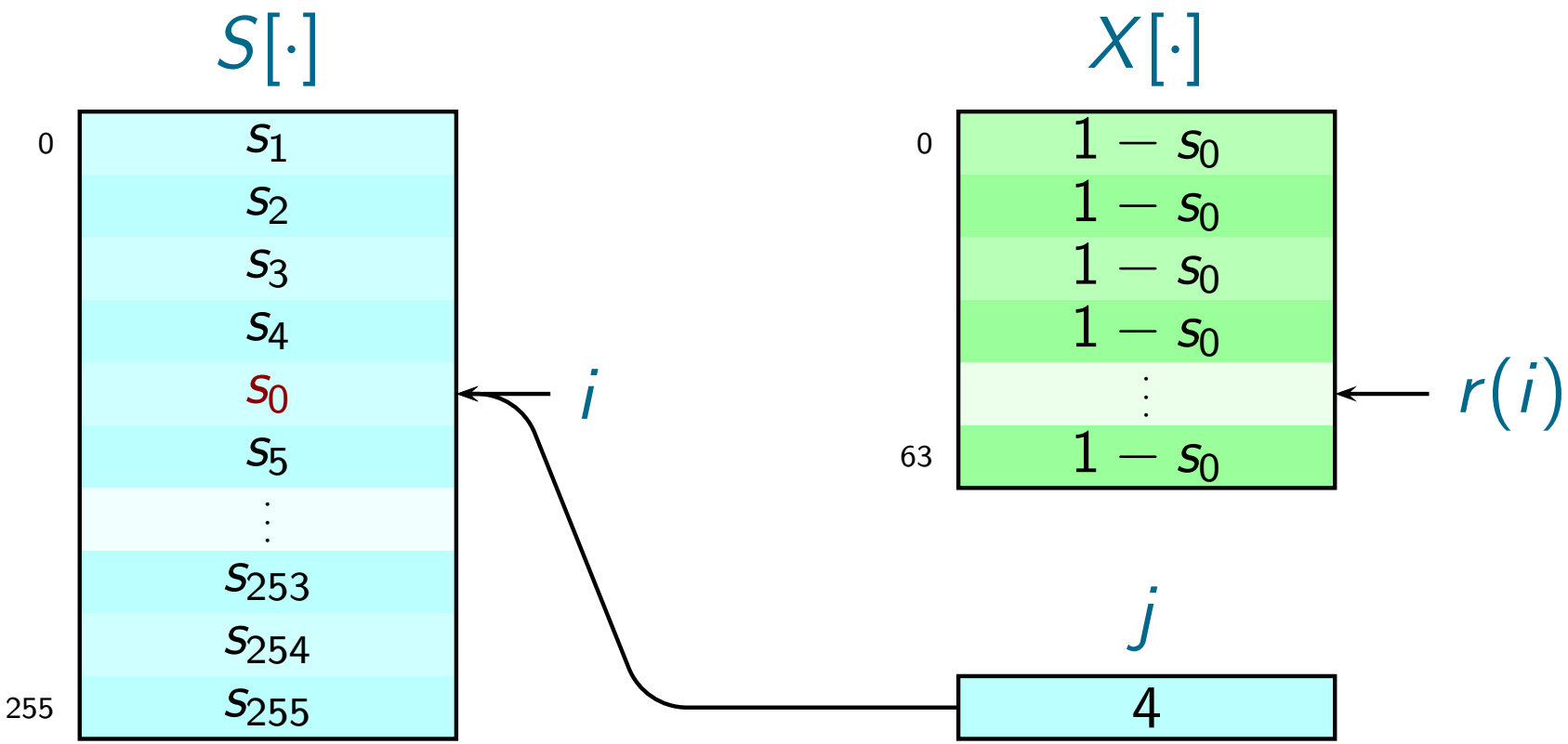

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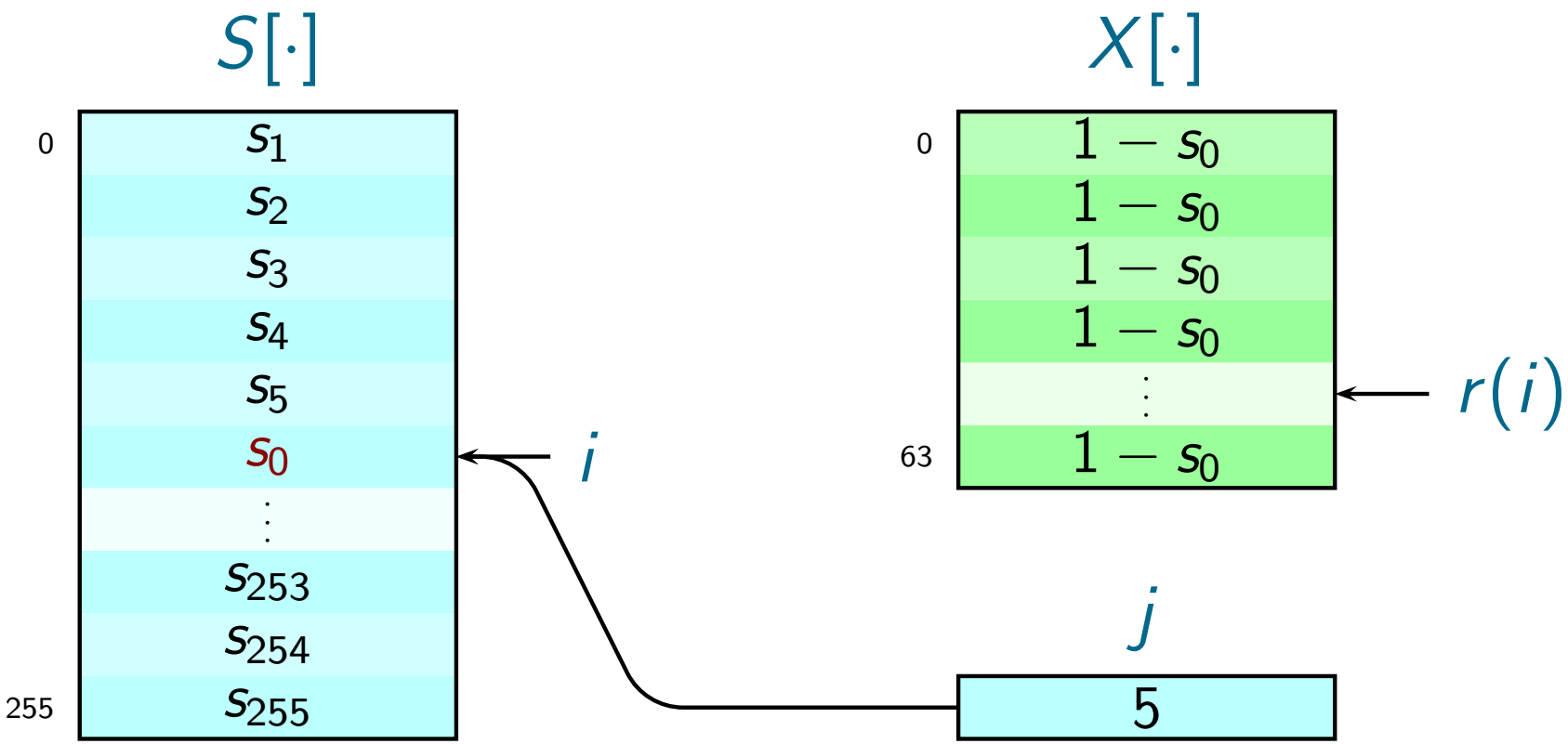

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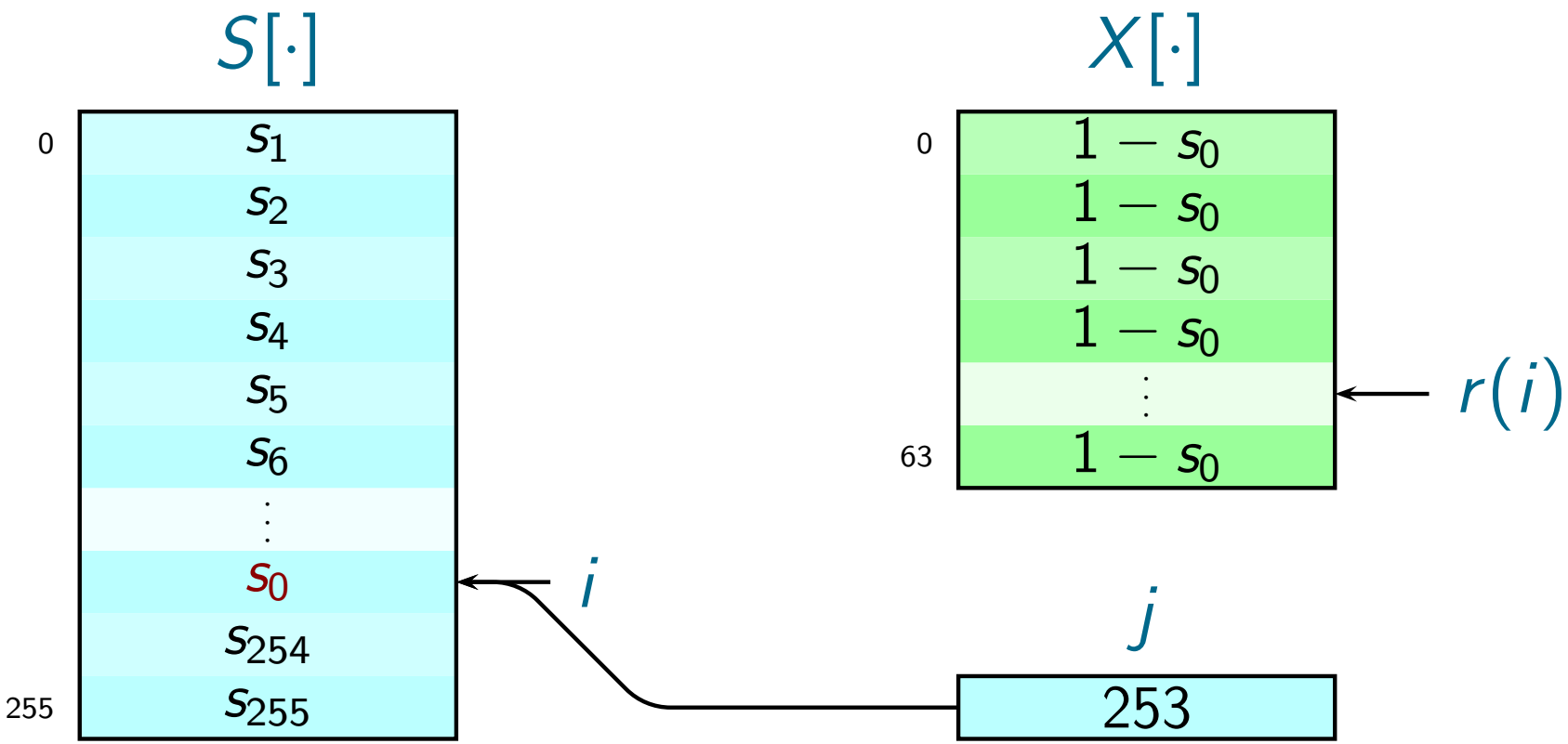

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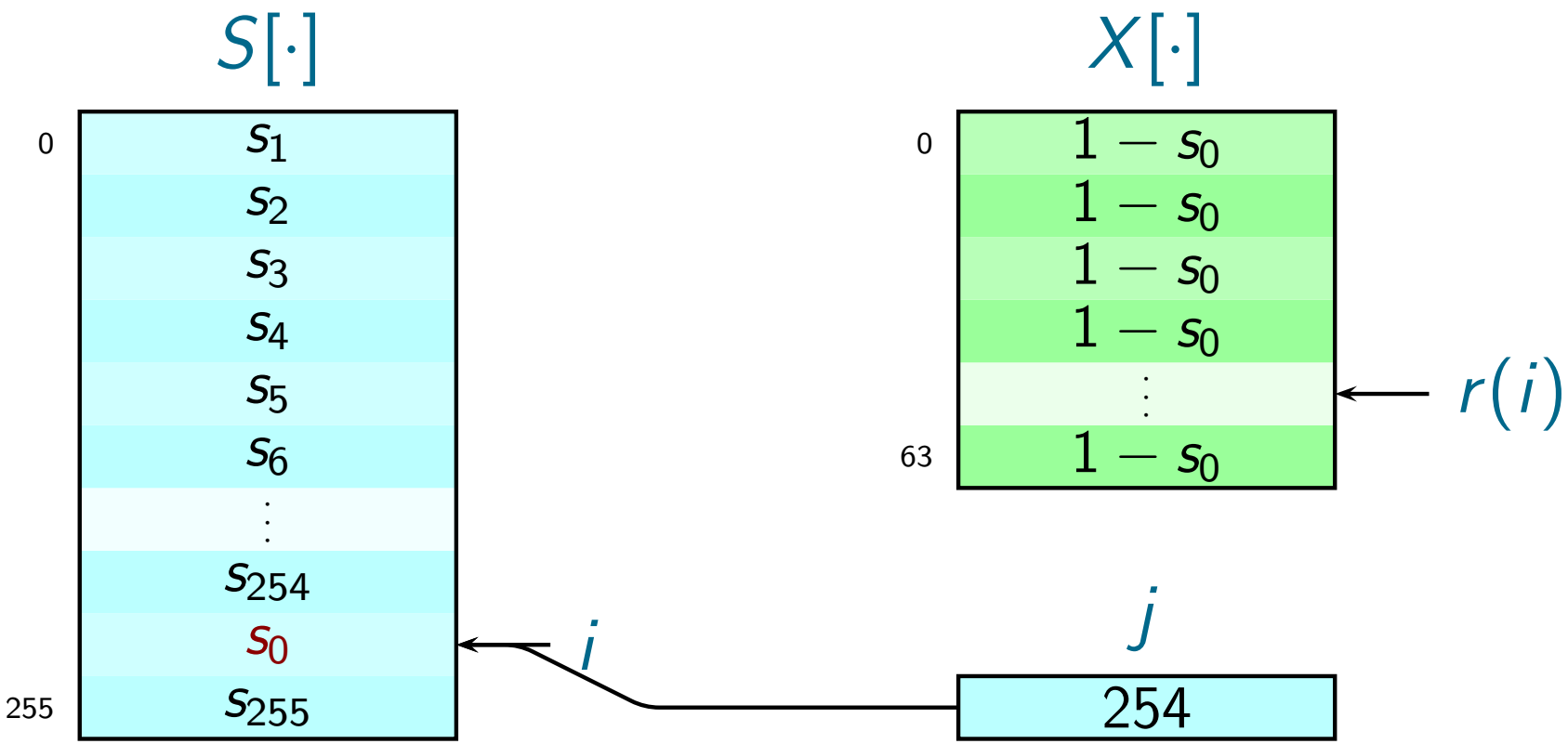

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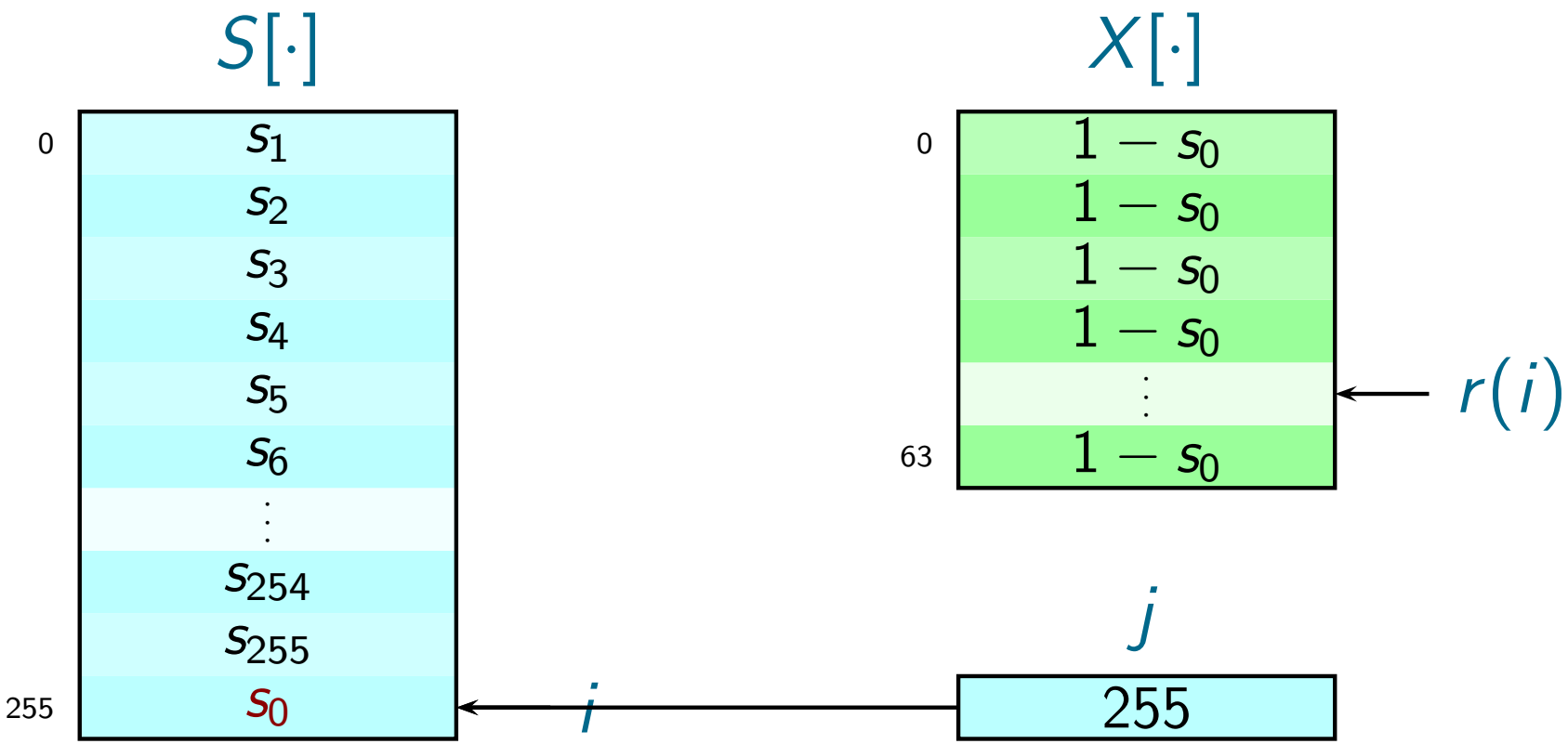

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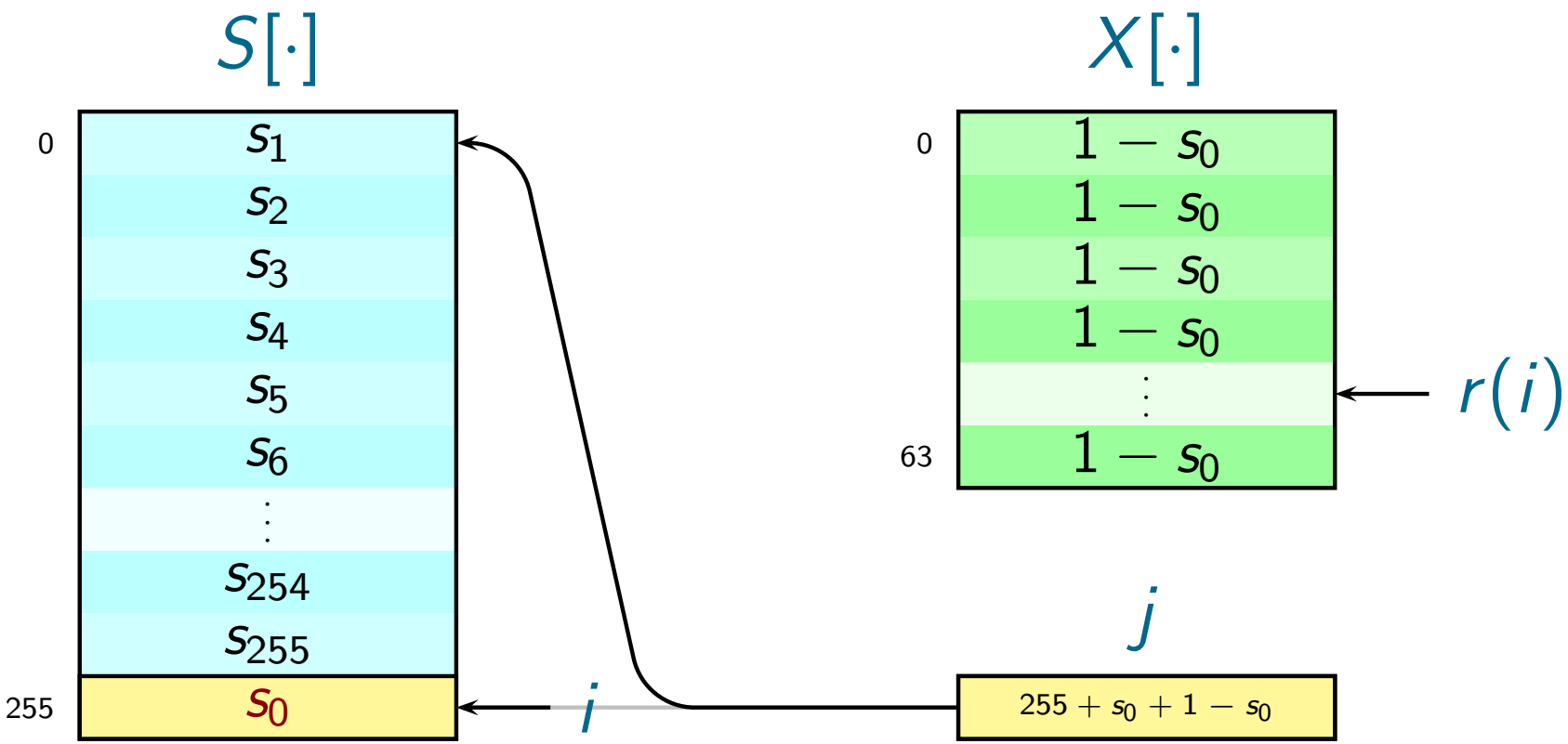

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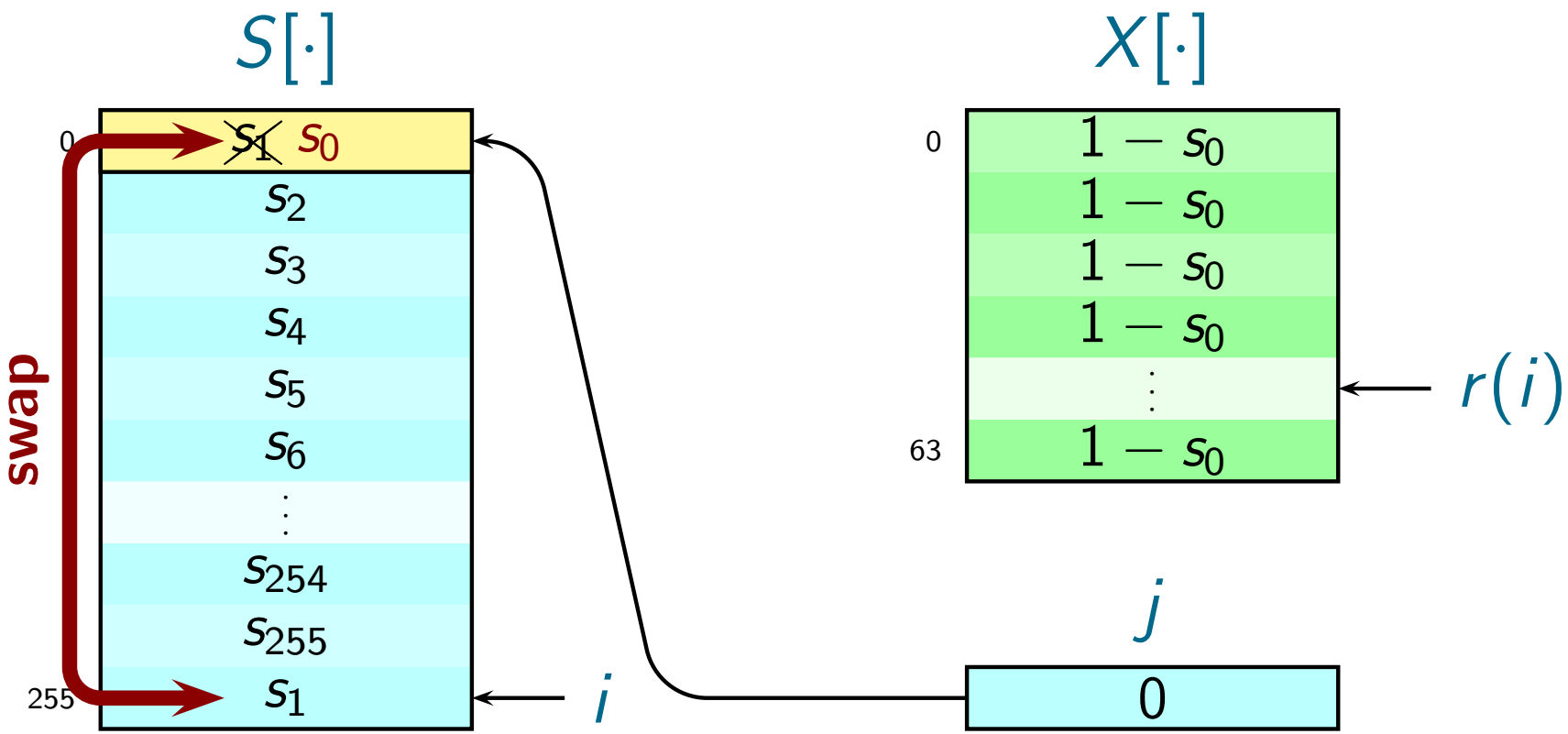

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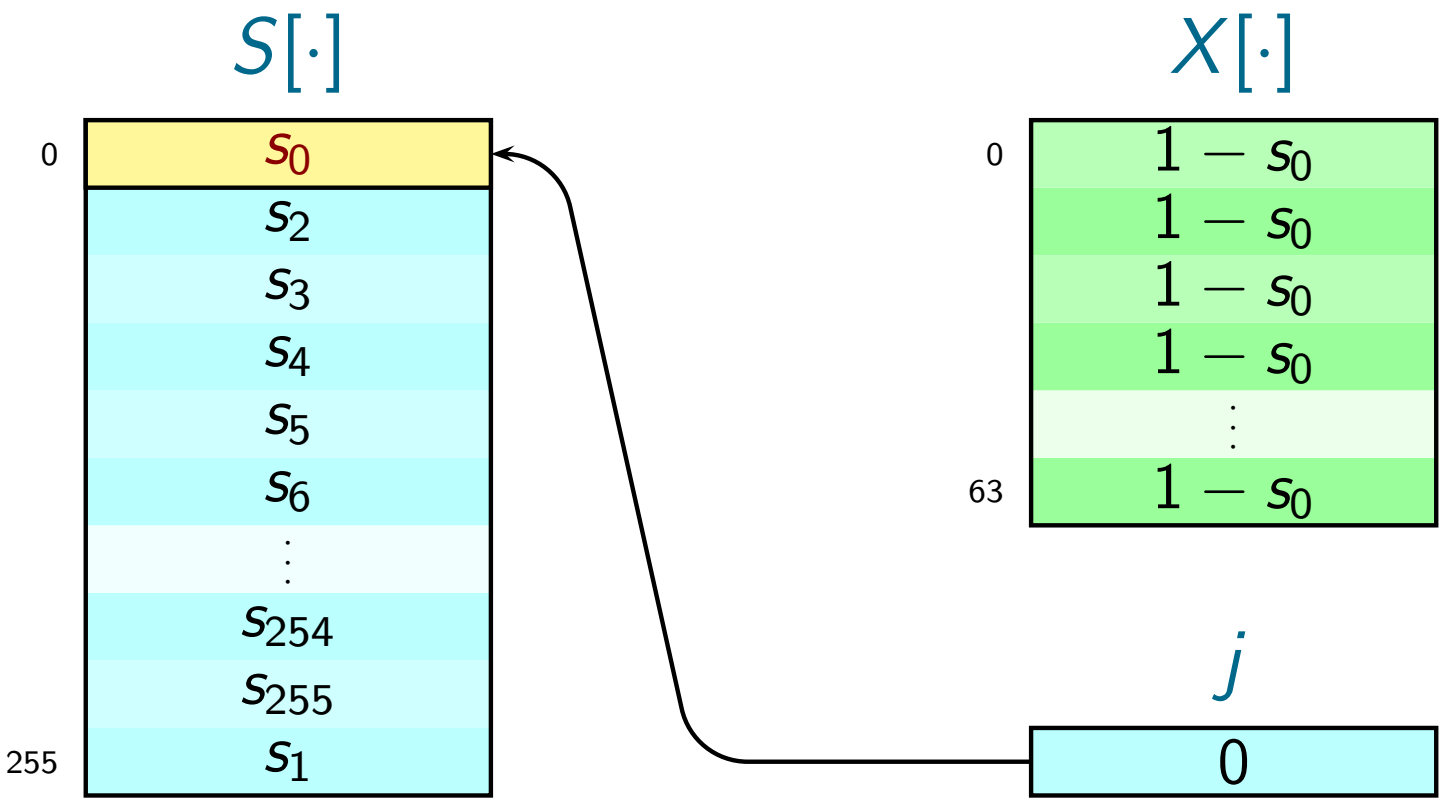

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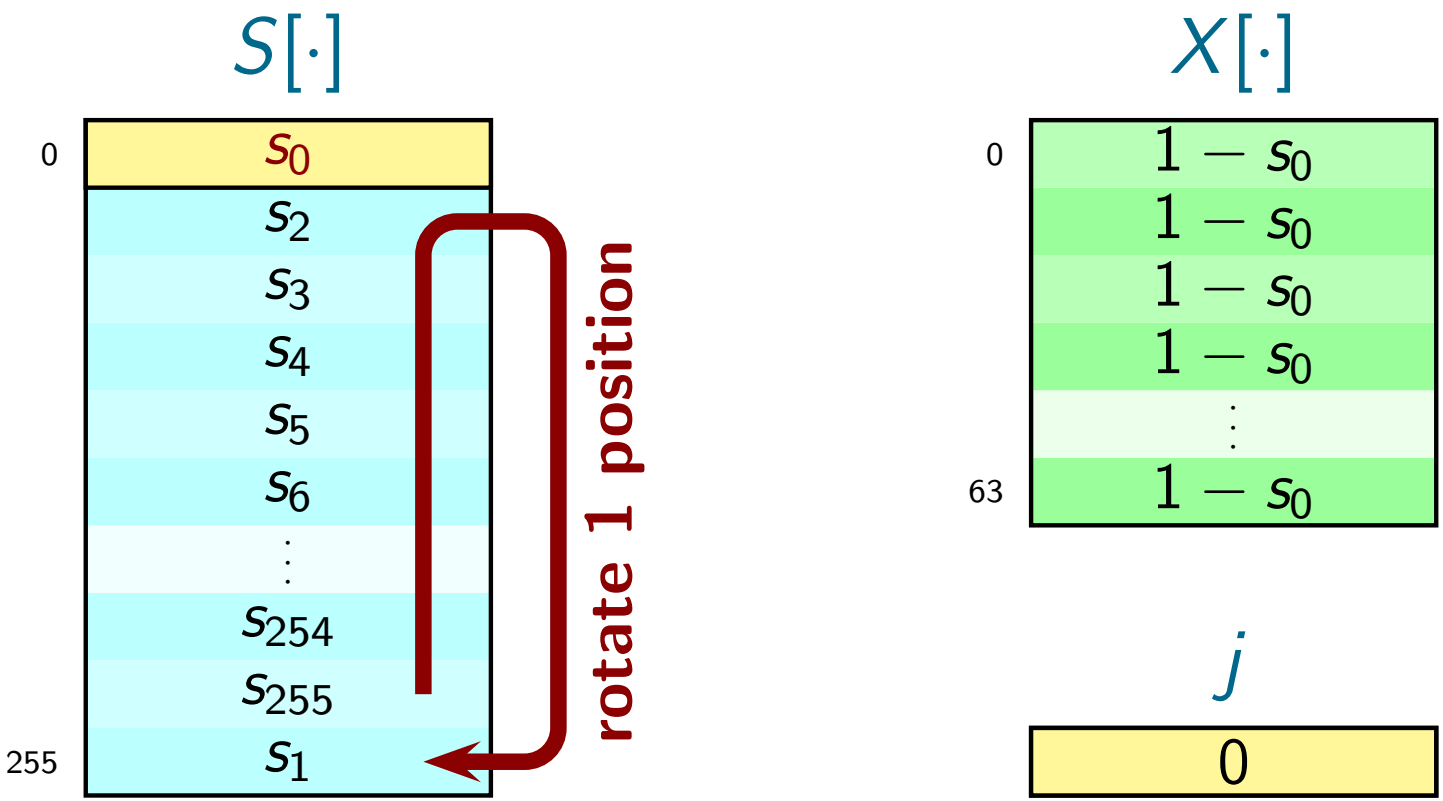
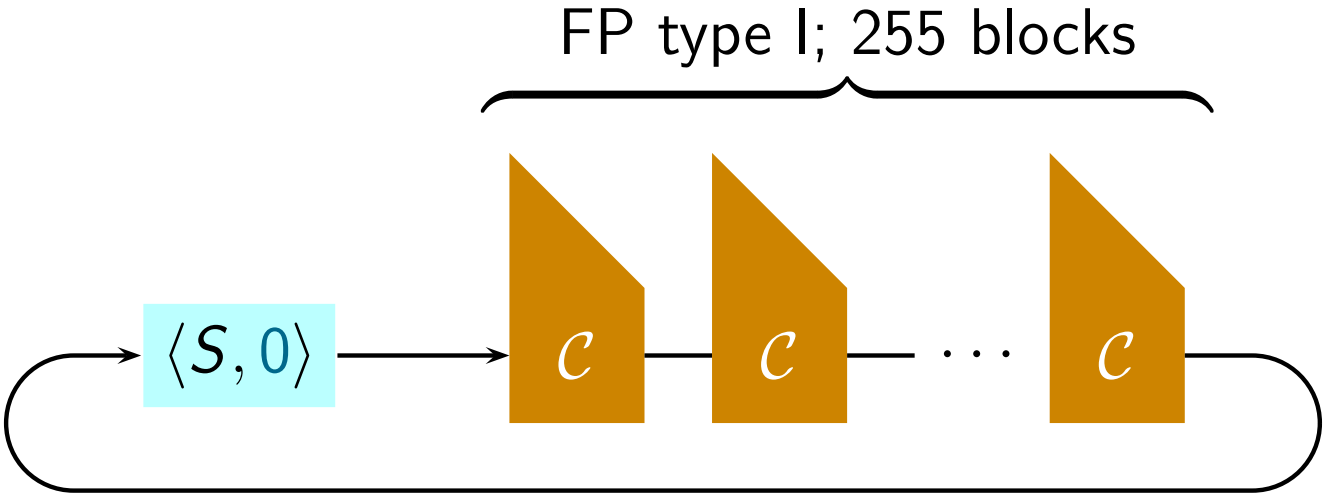


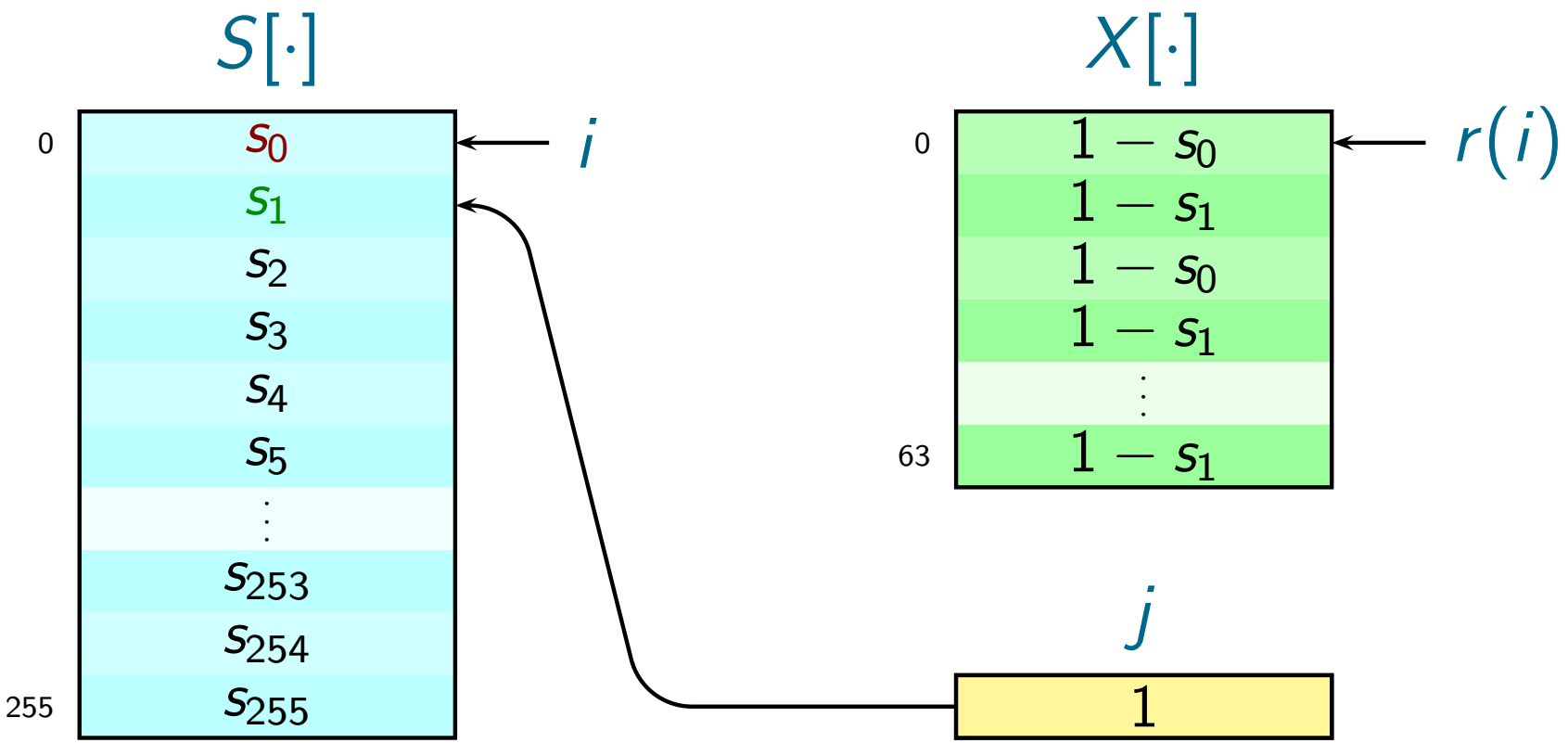
1: **for** $i = 0$ to 255 **do**
2: $\quad j \leftarrow j + S[i] + X[r(i)] \bmod 256$
3: $\quad$ swap $S[i], S[j]$
4: **end for**

Fixed Points of Type I

- Repeat **255 times** partial state rotation of type I
- **Only condition**: $j = 0$
- **Cost?** One subtraction mod256



Partial State Rotations of Type II



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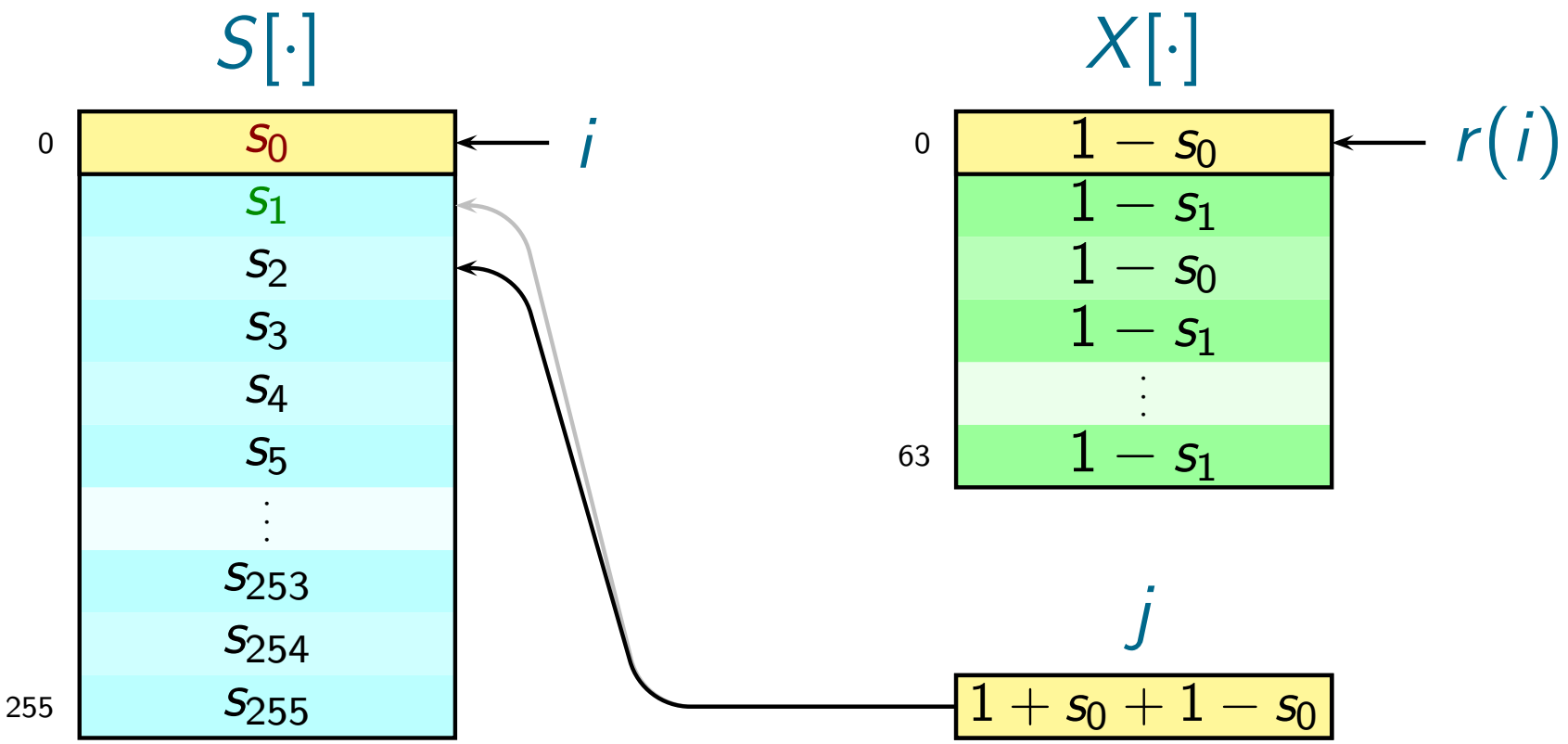

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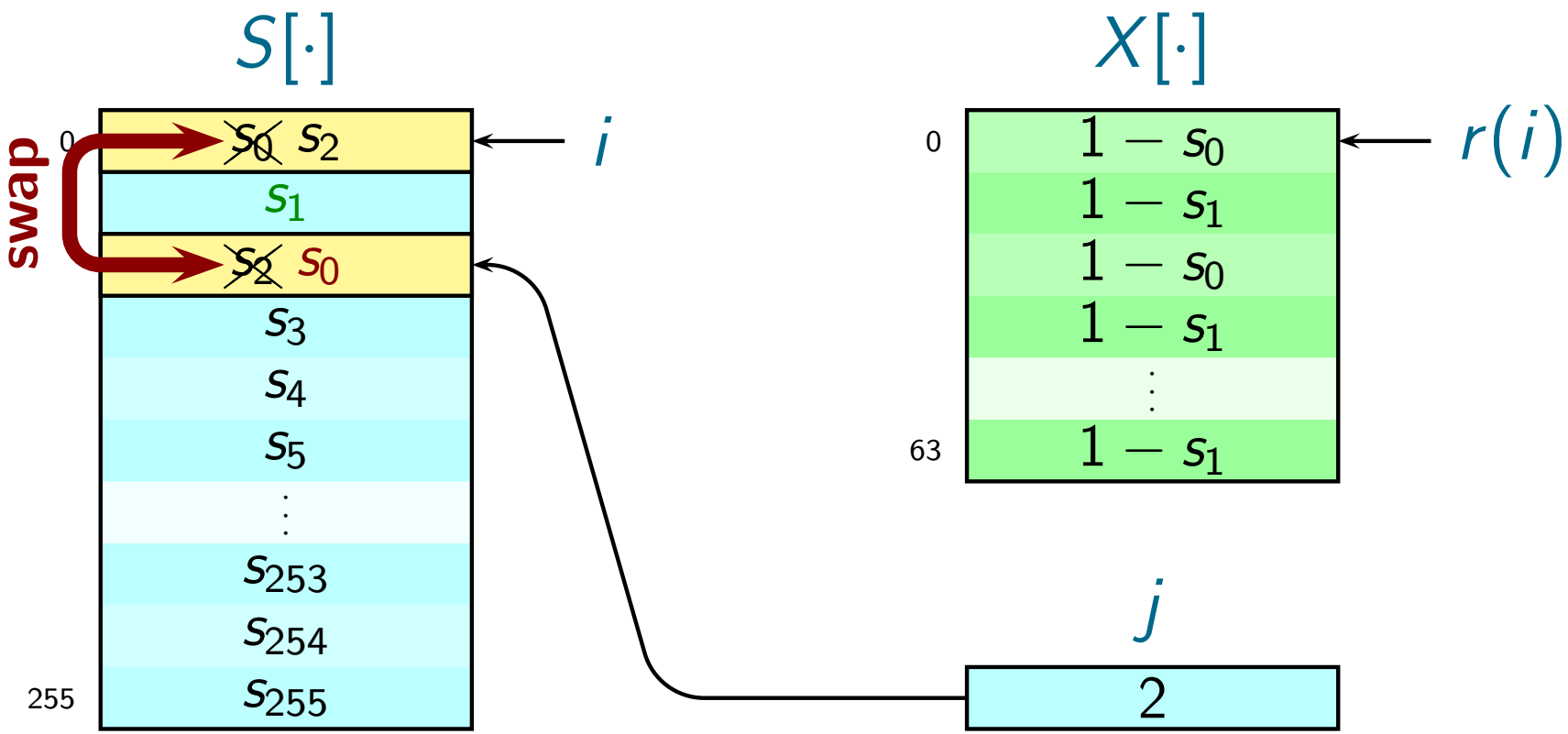

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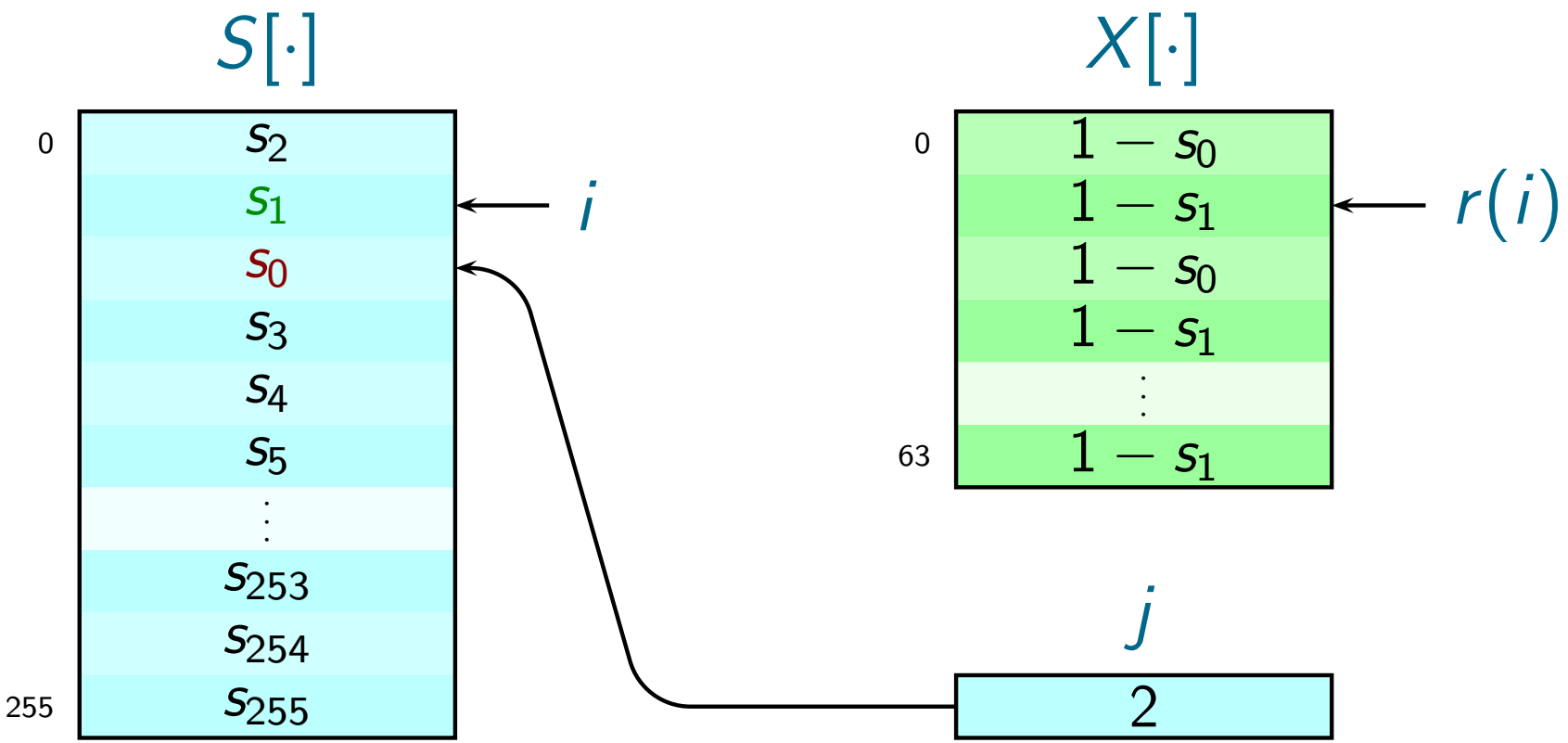

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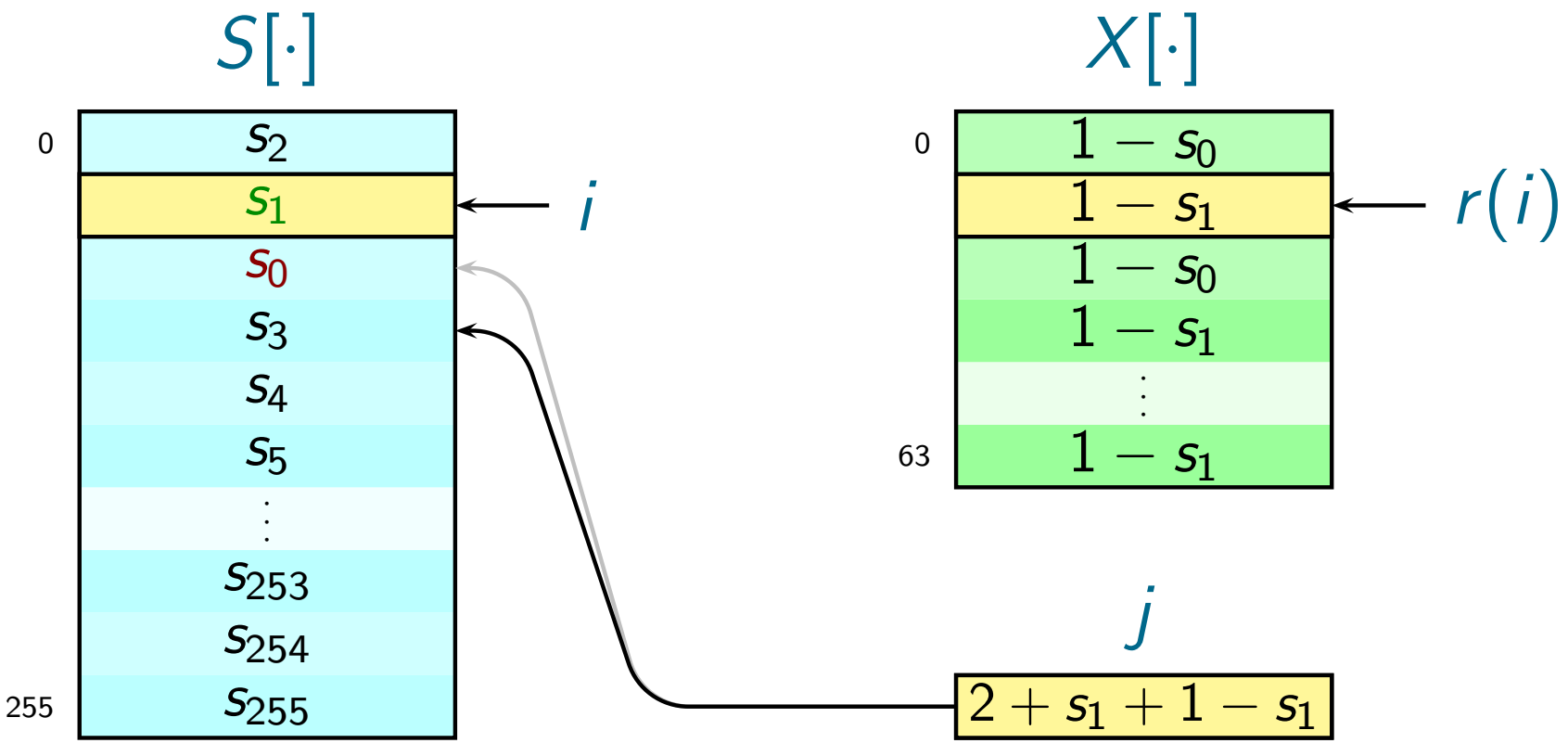

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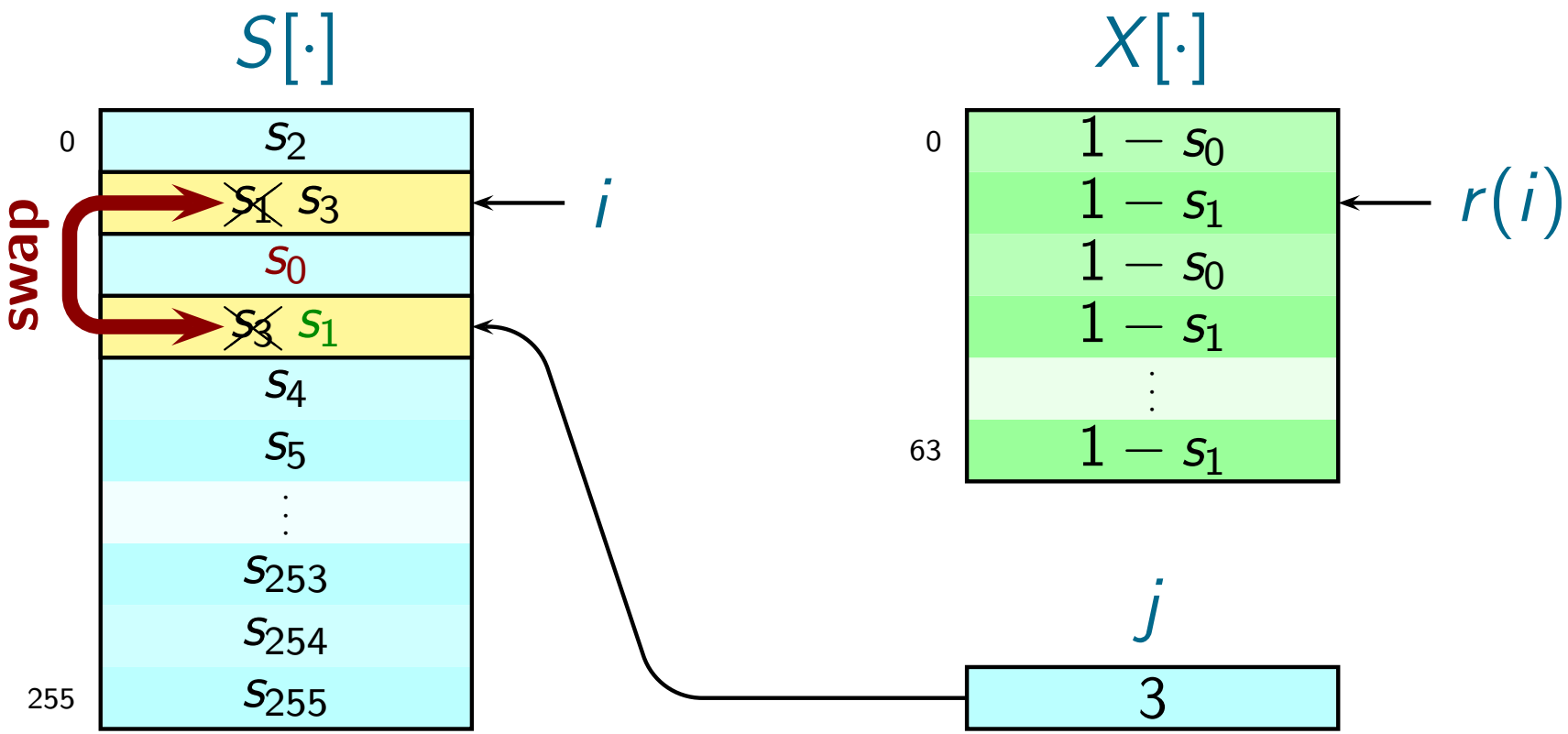

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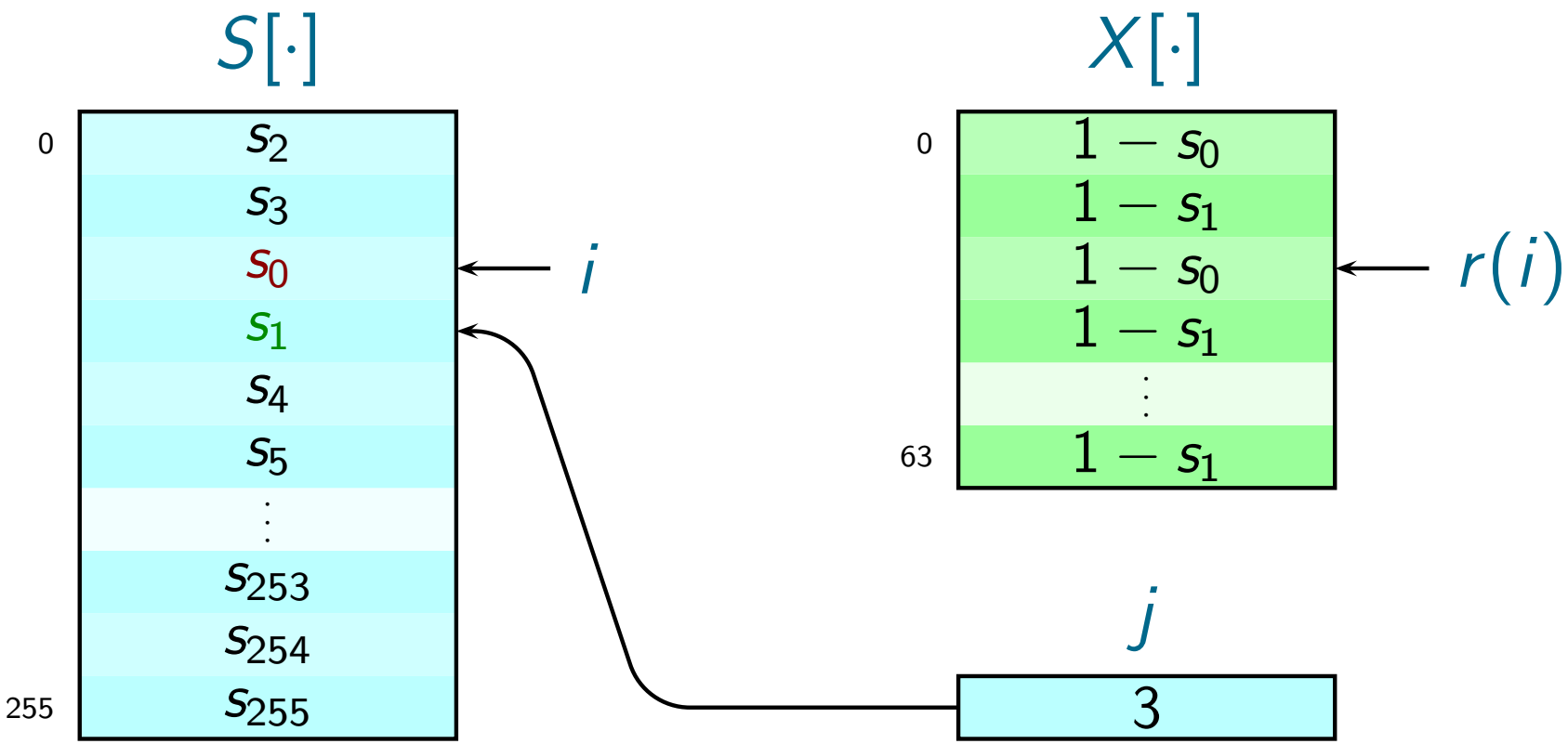

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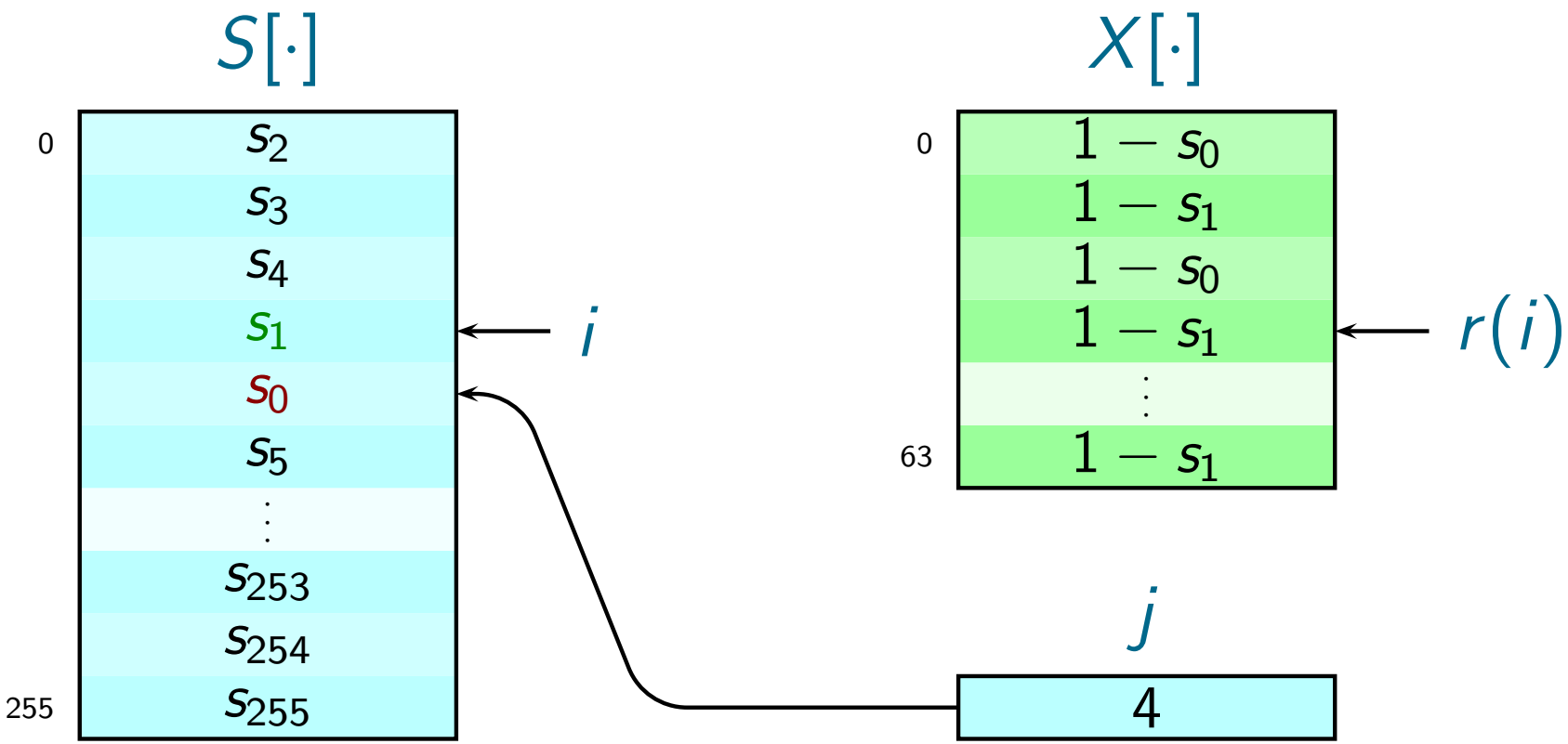

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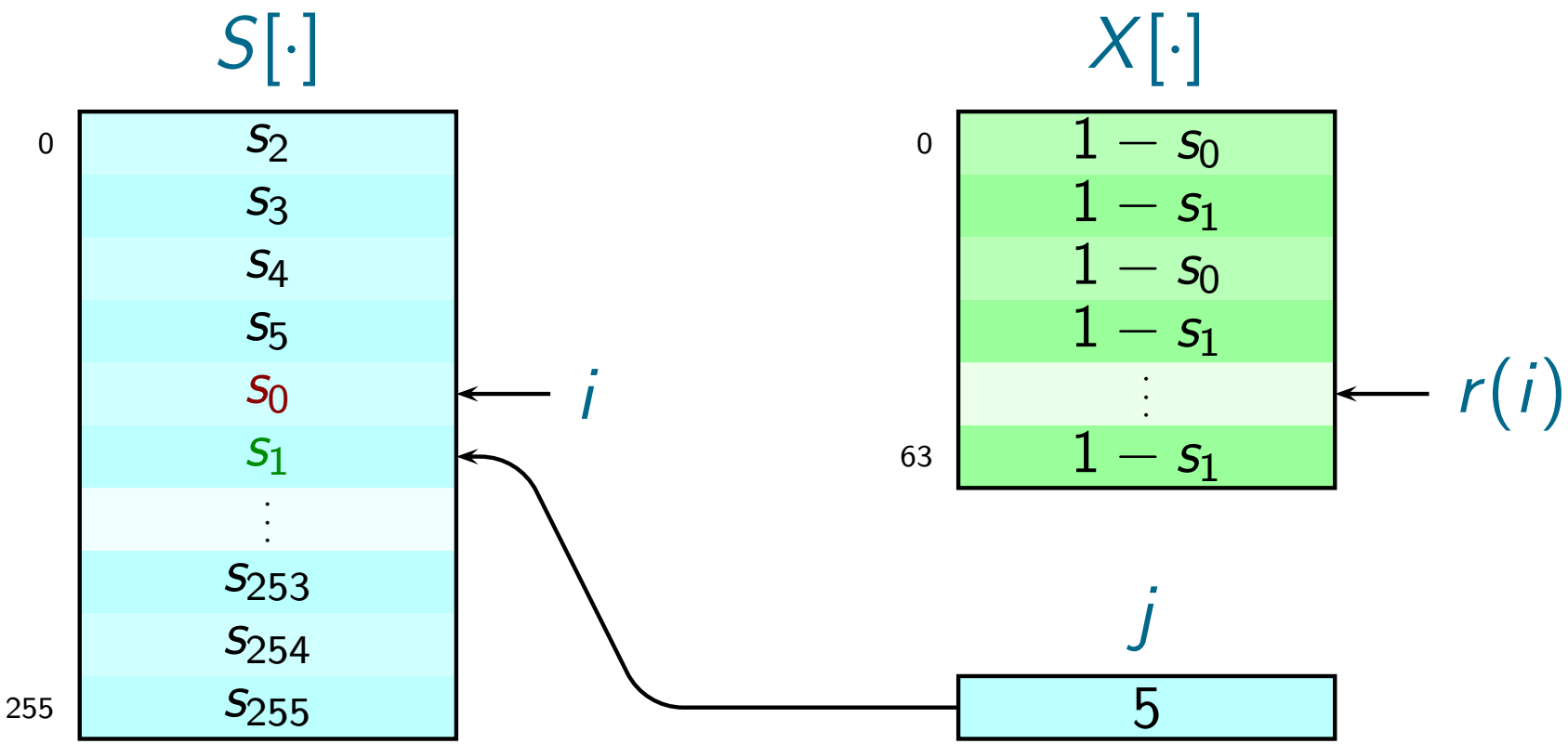

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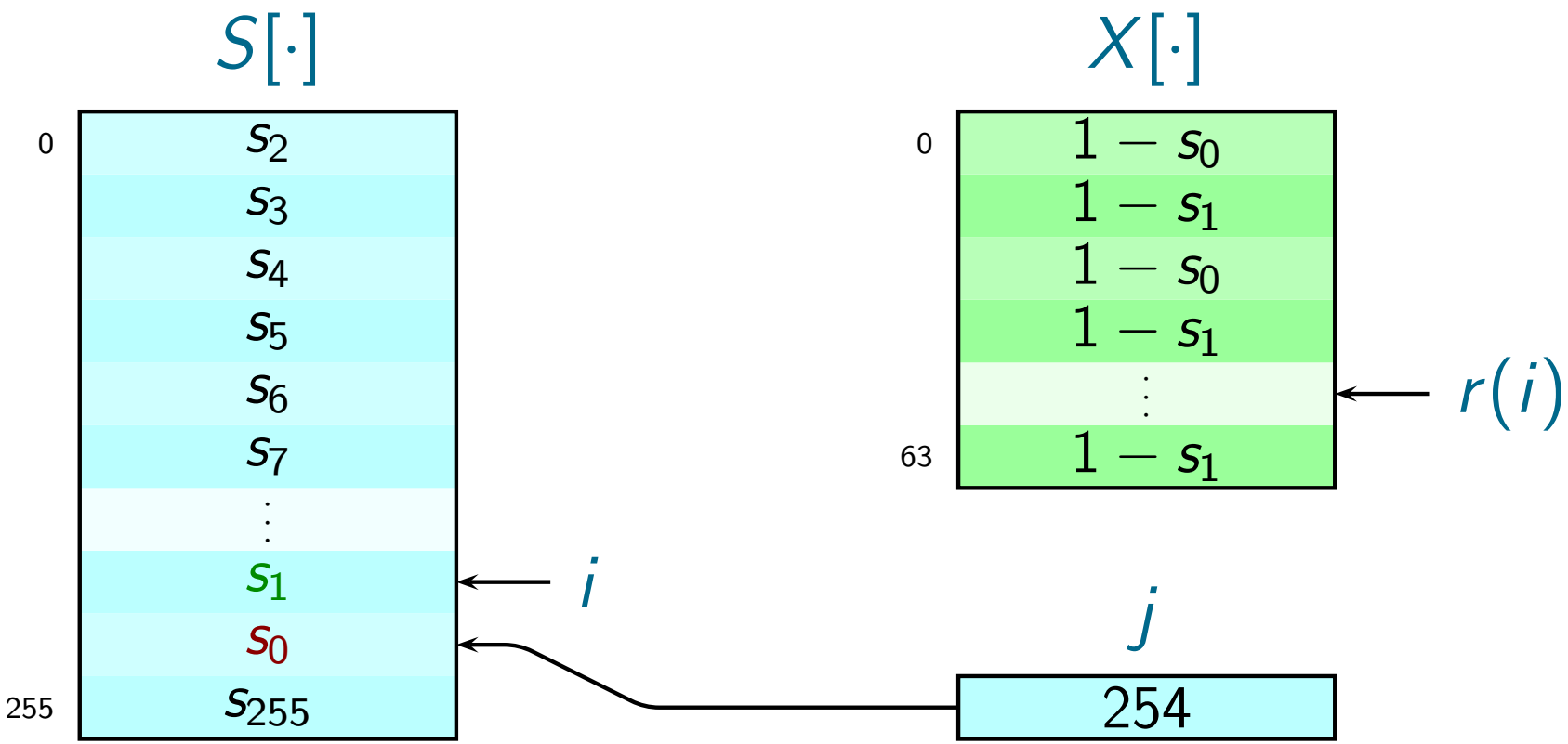

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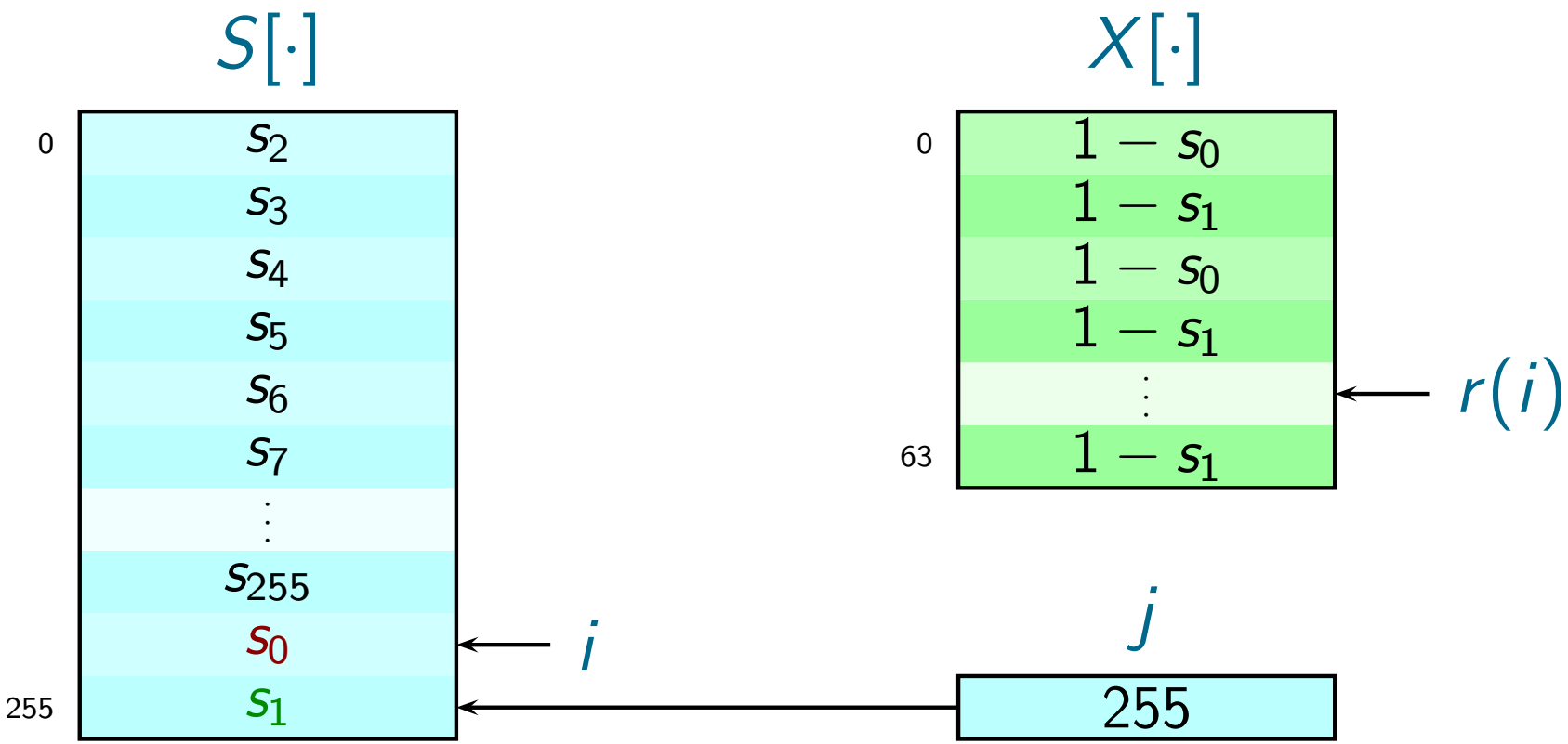

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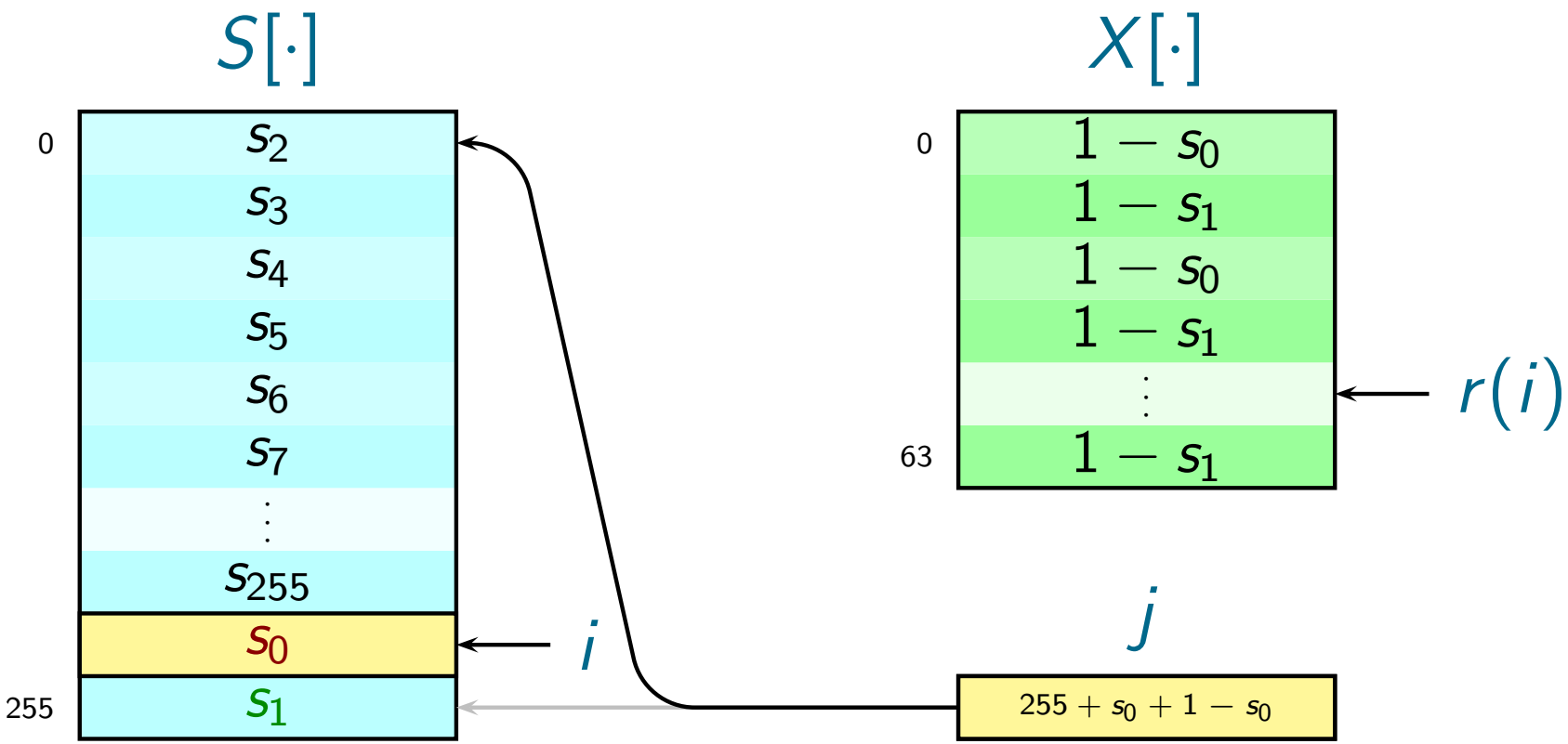

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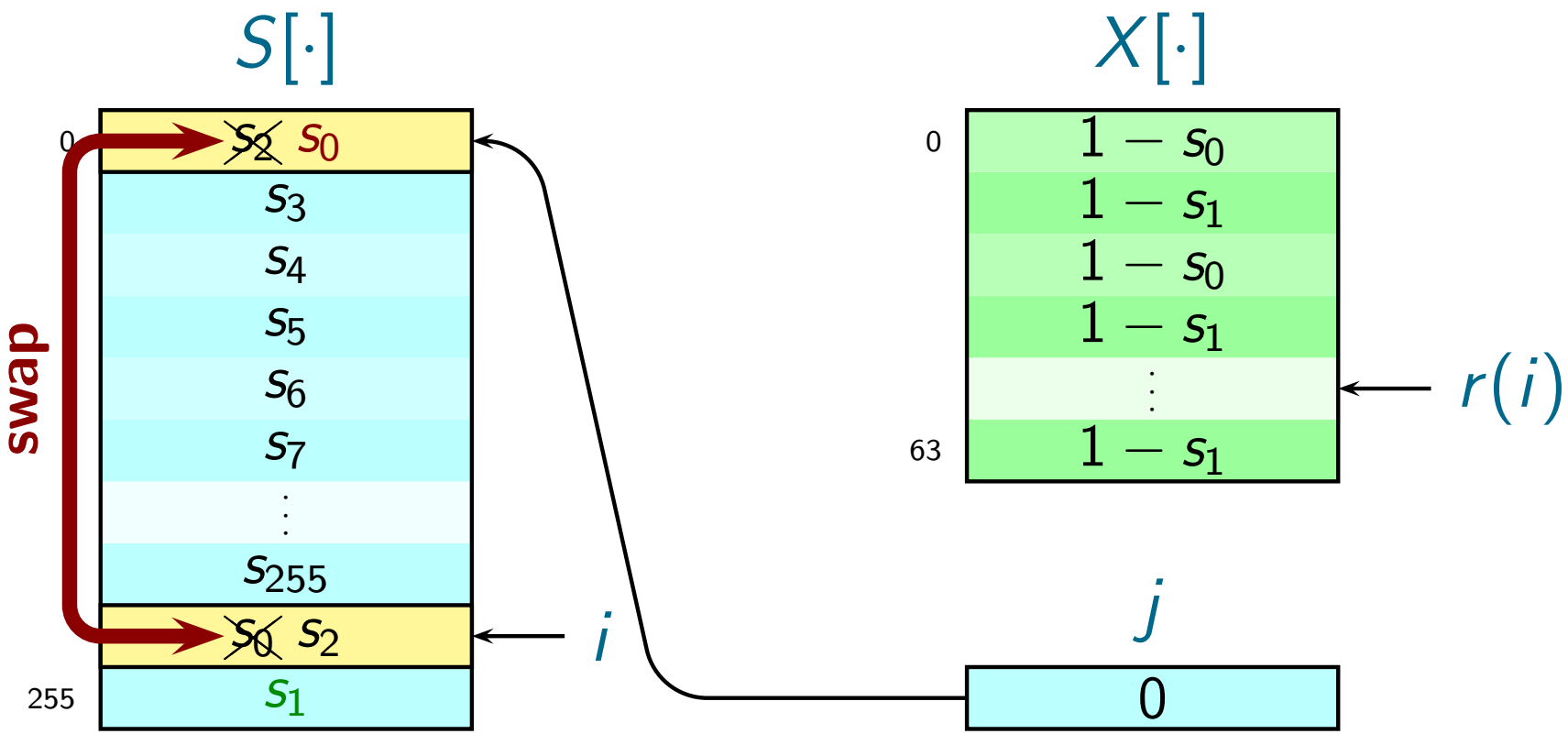

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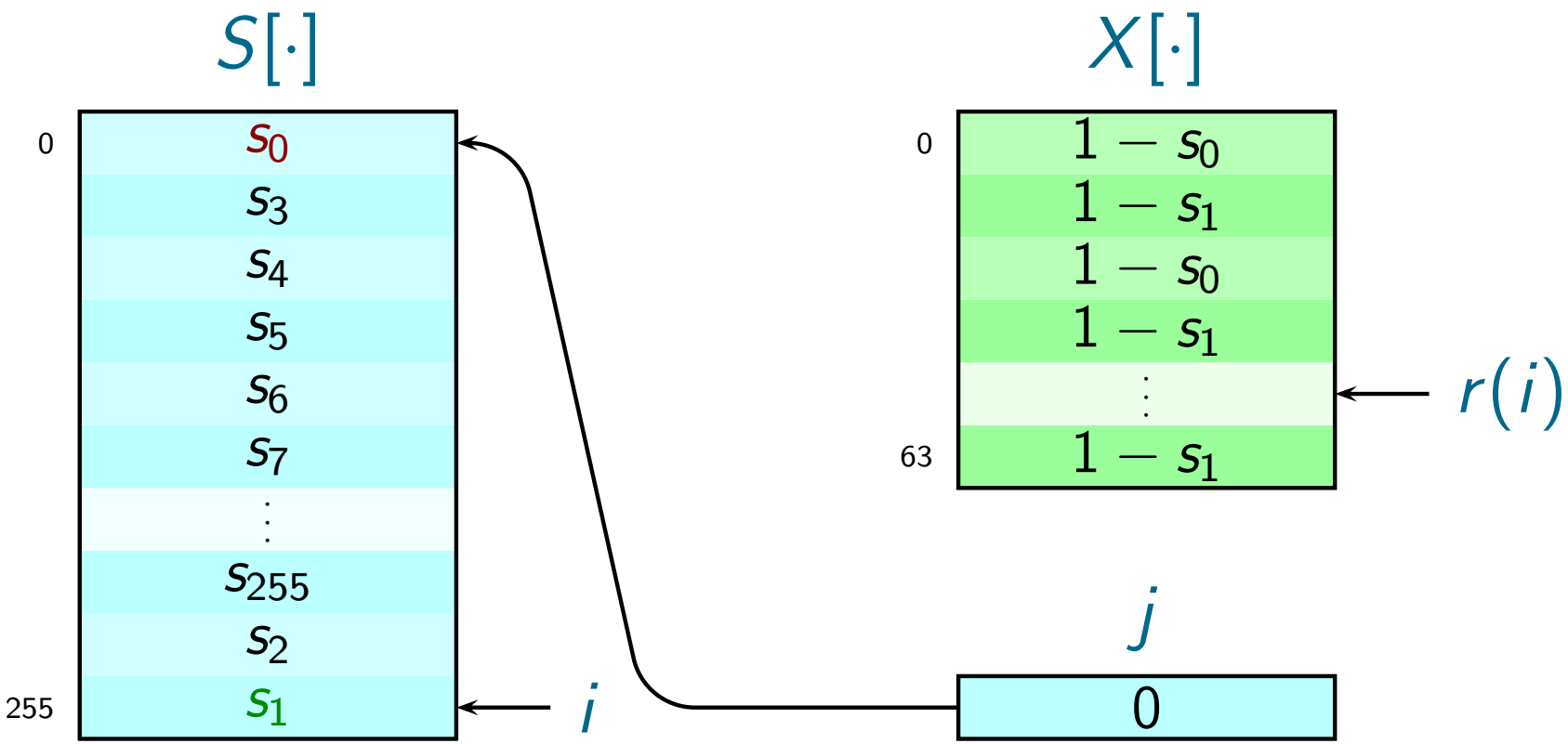

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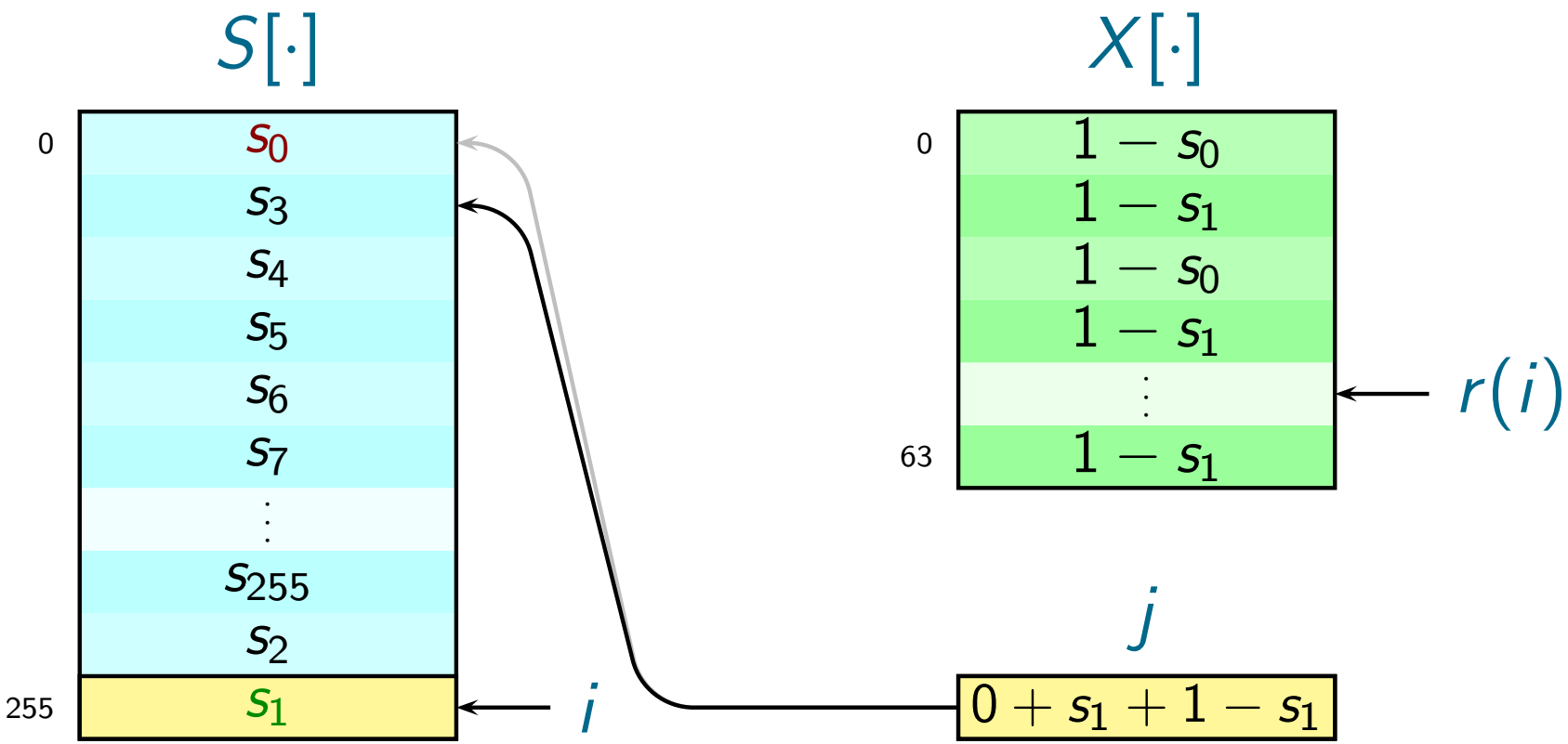

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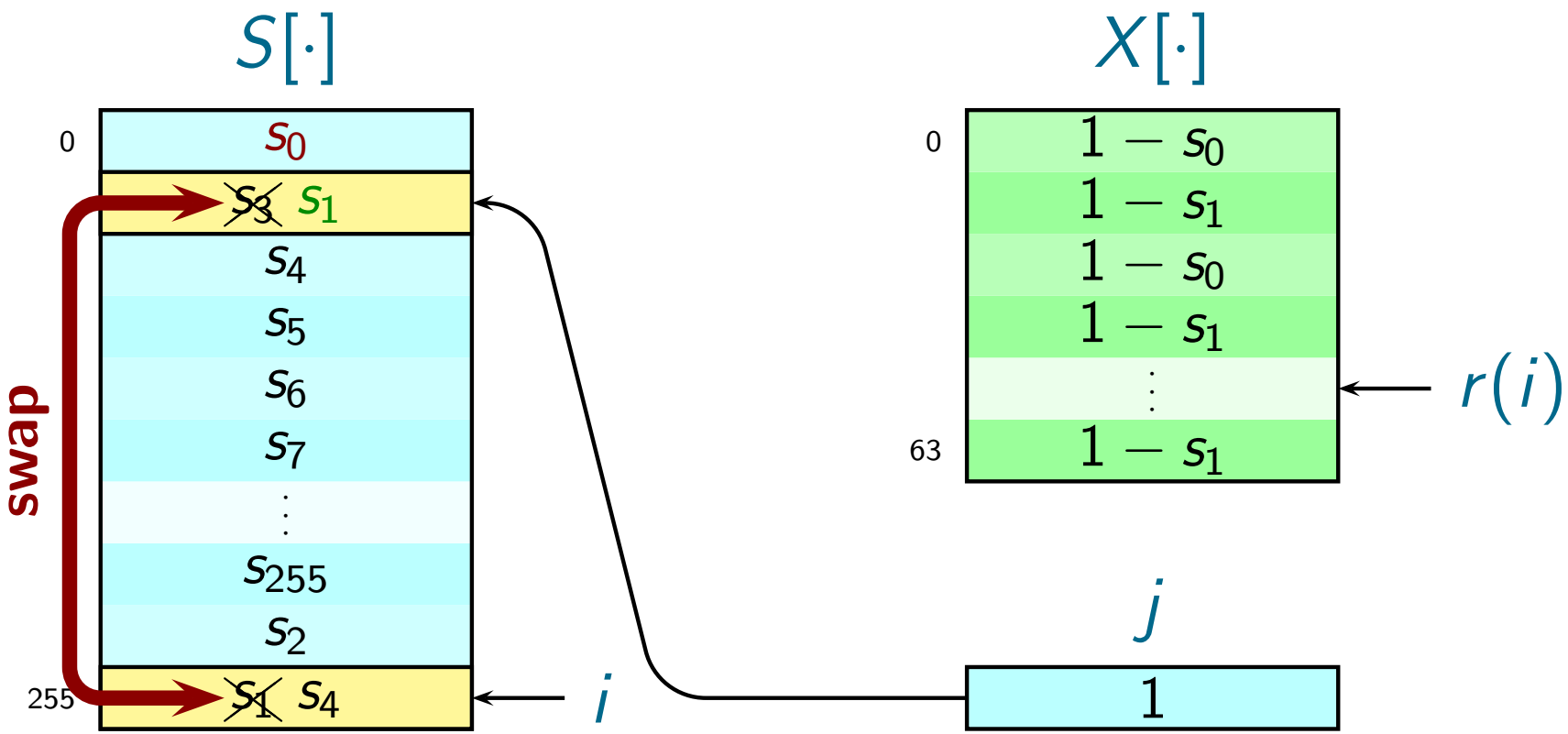

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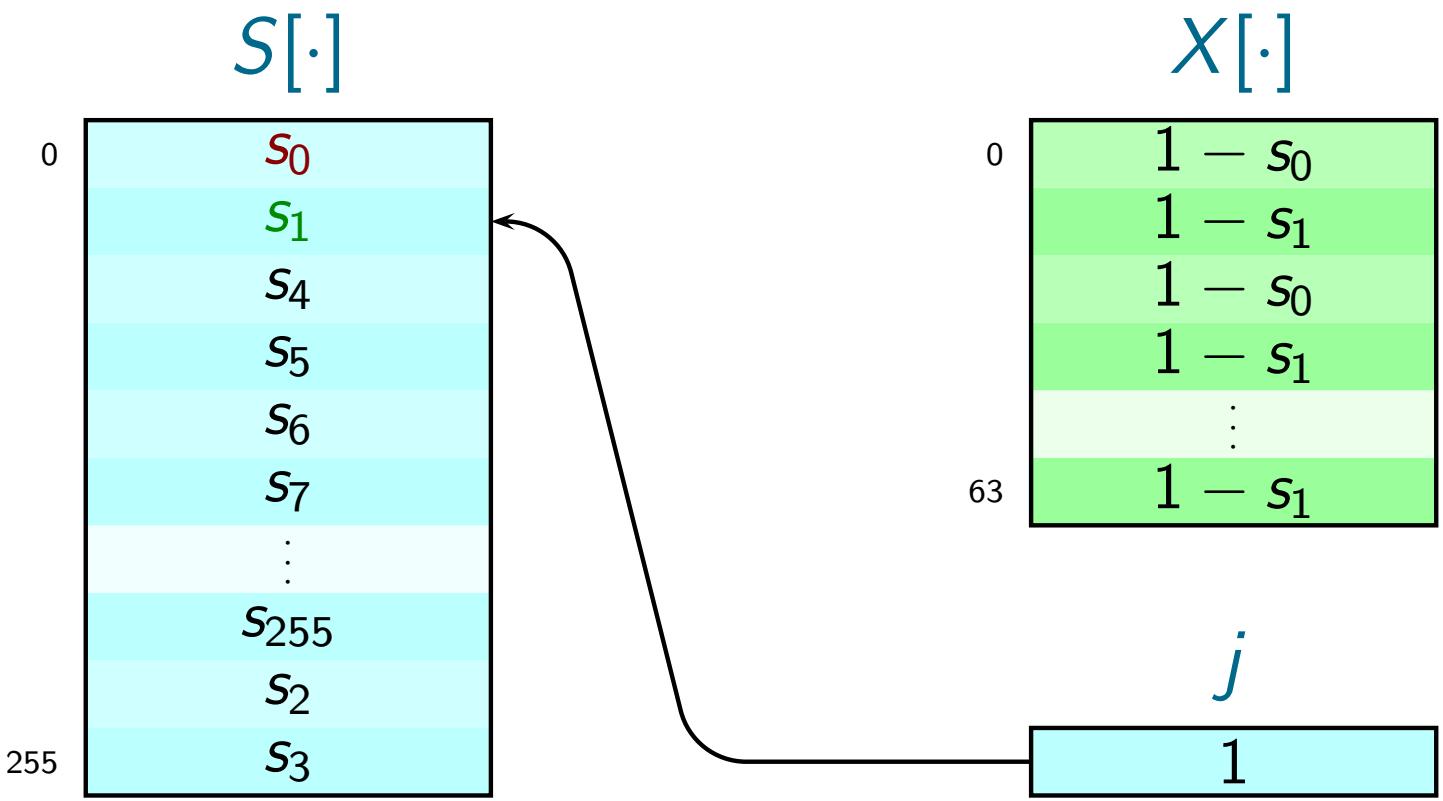

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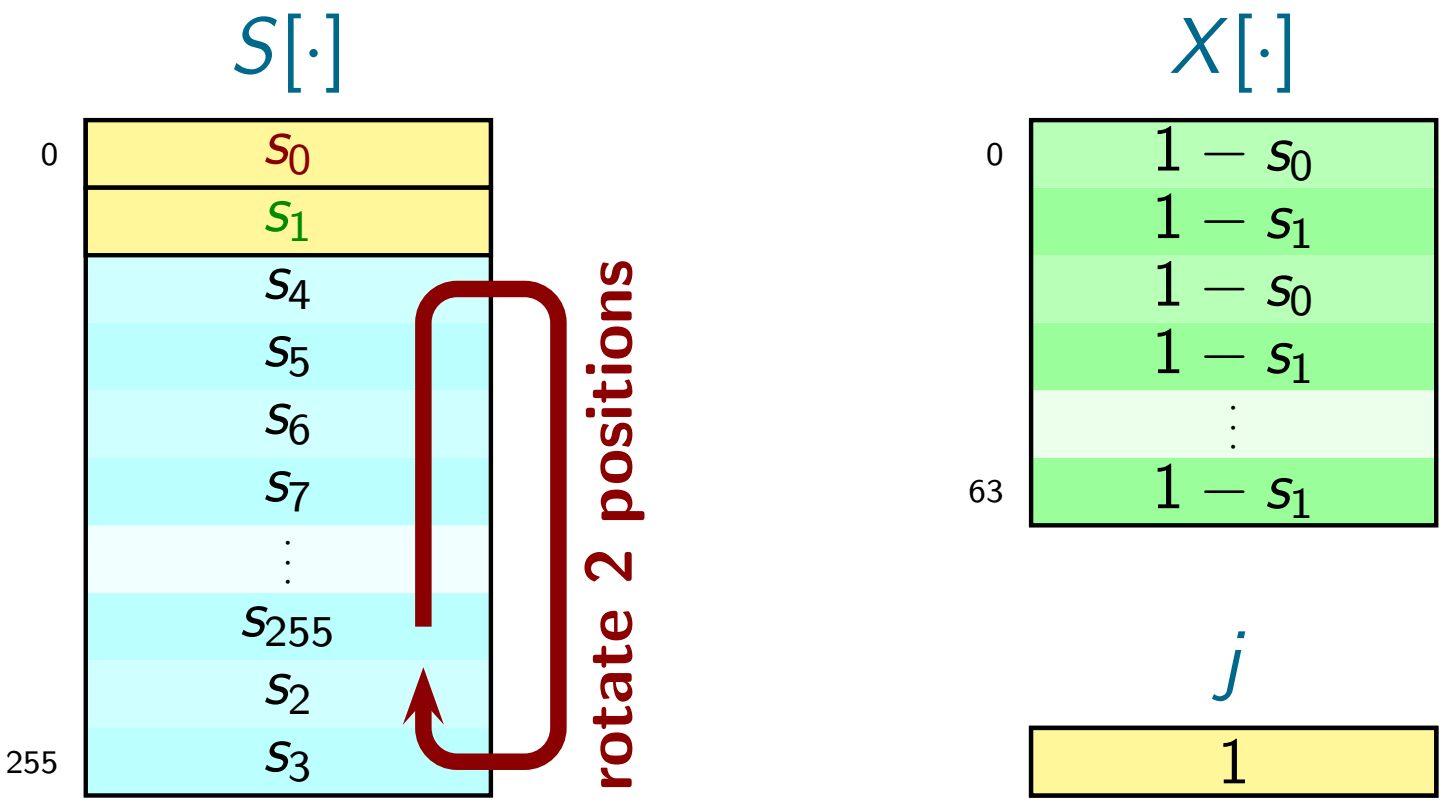

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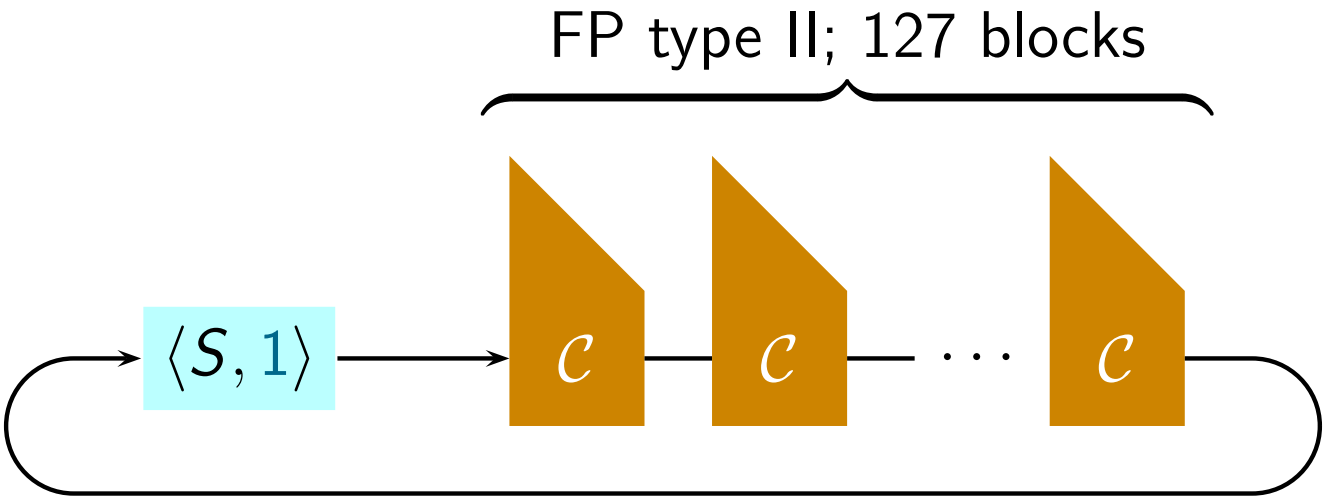
Fixed Points of Type II

- Repeat **127 times** partial state rotation of type II
- **Only condition**: $j = 1$
- **Cost?** Two subtractions mod256



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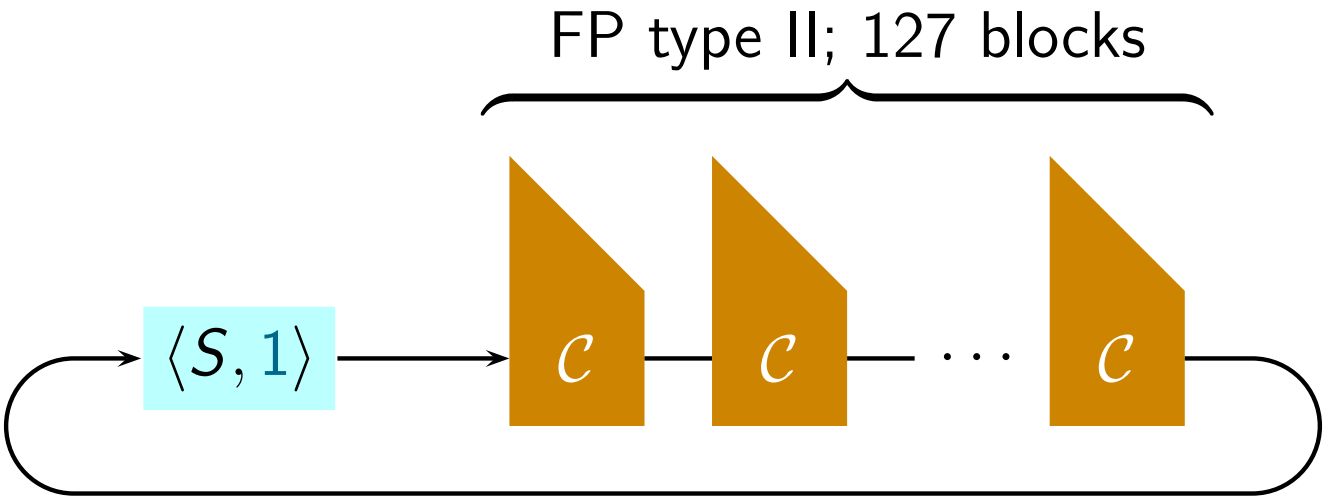


Type III and beyond?

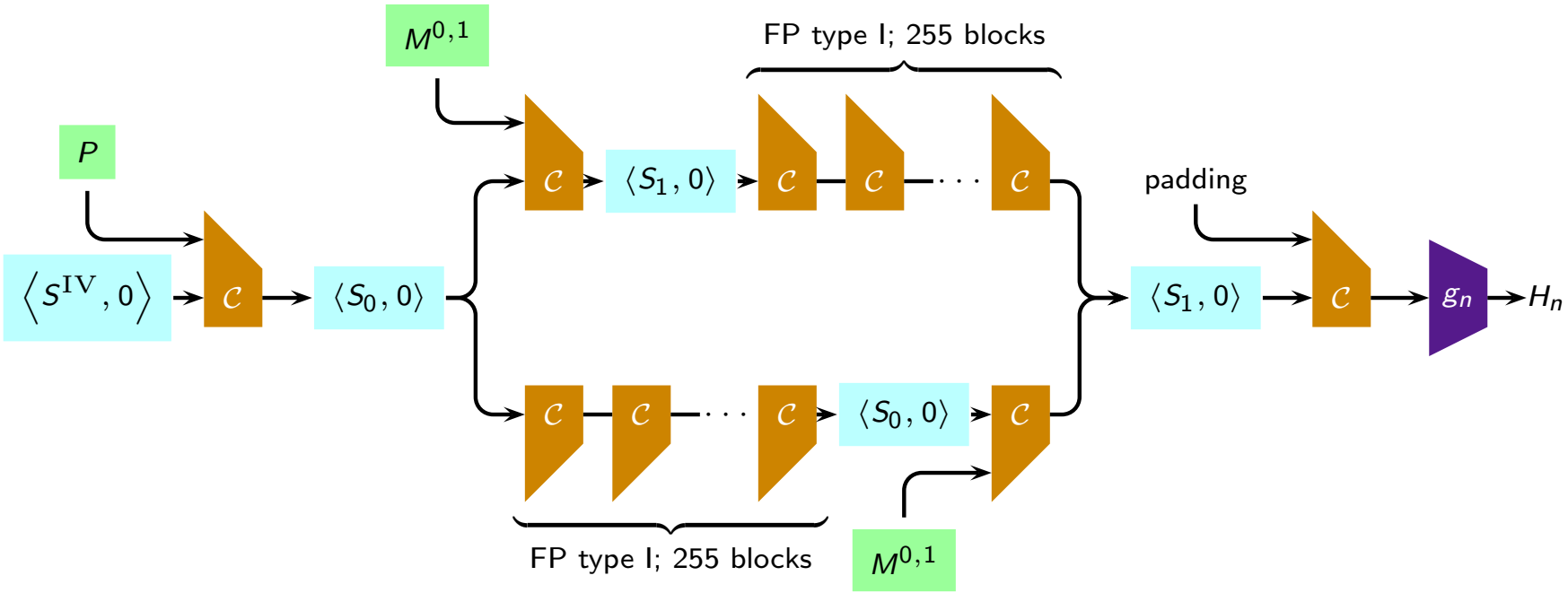
- **Not possible** due to message reordering $r(\cdot)$

Outline

1. Introduction
   - Cryptographic Hash Functions
   - RC4-Hash
2. Fixed Points for RC4-Hash
   - Type I
   - Type II
3. **Collisions for RC4-Hash**
4. Conclusion

Collisions for RC4-Hash

Using Fixed Points of Type I

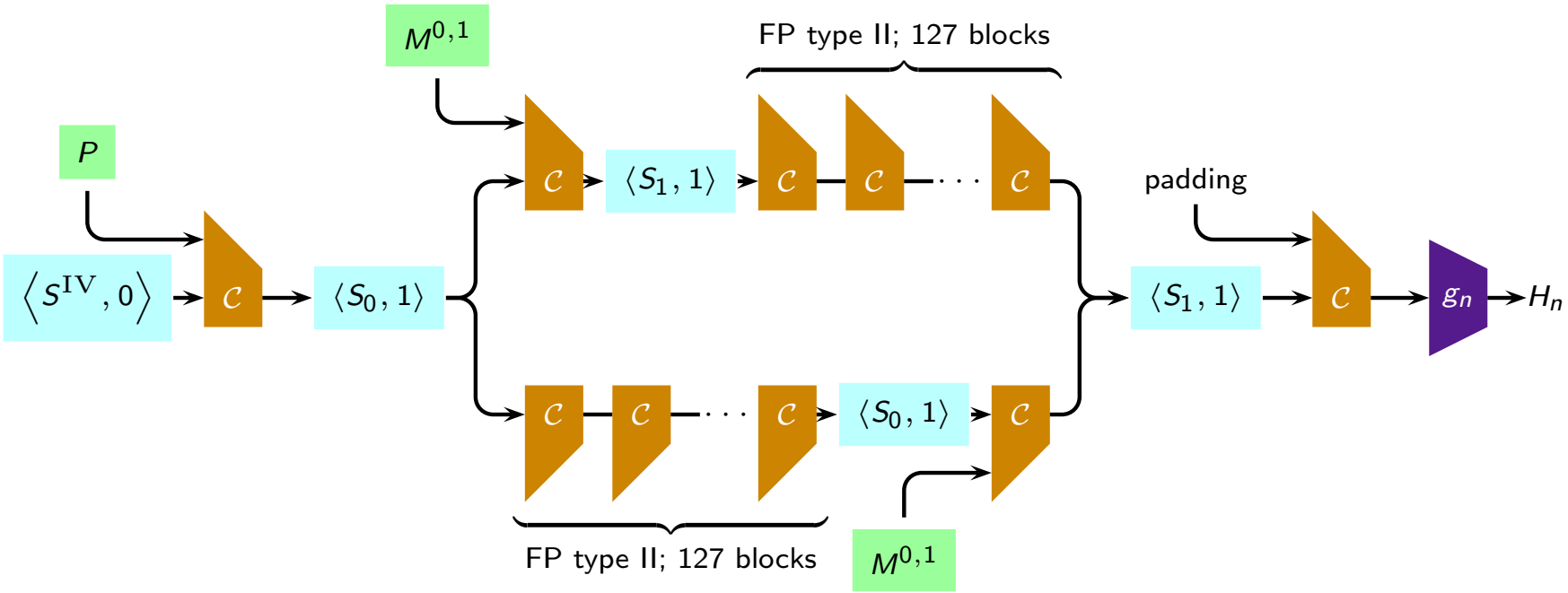


Time Complexity $\approx 2^9$ evaluations of $\mathcal{C}$

- Prefix block P $\approx 2^8$
- Message block $M^{0,1}$ $\approx 2^8$

Collisions for RC4-Hash

Using Fixed Points of Type II



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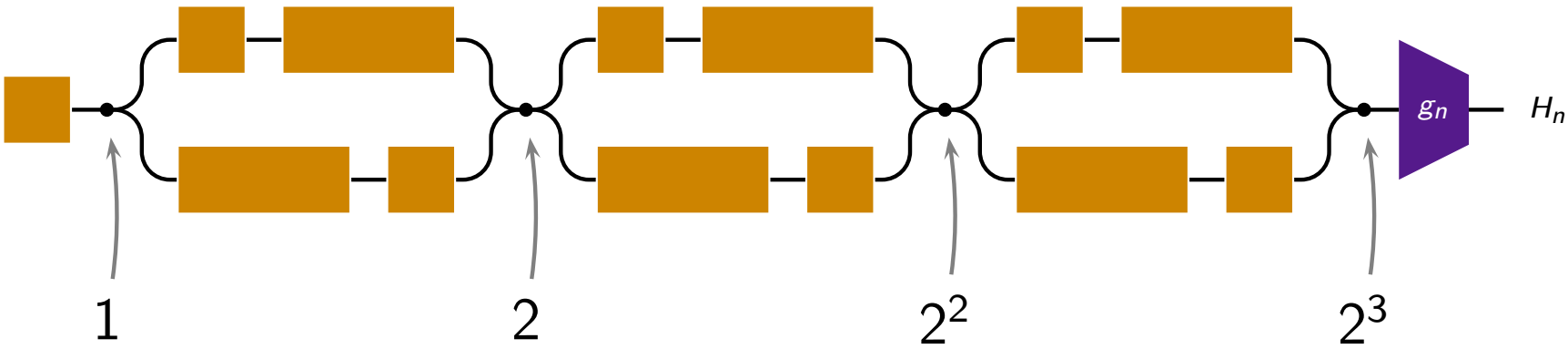
Collisions for RC4-Hash

Multicollisions



Multicollisions

- 2^k colliding messages
- Technique of A. Joux
- Time complexity $\approx 2^7 + k \cdot 2^8$

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Conclusion

RC4-Hash is not collision resistant

- Easy construction of **fixed points**
- Practical **collision attack**
  - Time complexity $\approx 2^9$
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